THE NOVEMBER MEETING IN LOS ANGELES

The three hundred seventy-fourth meeting of the American Mathematical Society was held at the University of California, Los Angeles, California, on Saturday, November 23, 1940. The attendance was about one hundred including the following fifty members of the Society:

O. W. Albert, H. M. Bacon, Harry Bateman, Clifford Bell, H. F. Blichfeldt, J. L. Brenner, J. G. Charney, Myrtie Collier, P. H. Daus, T. C. Doyle, G. C. Evans, A. L. Foster, H. E. Glazier, W. H. Glenn, E. C. Goldsworthy, H. J. Hamilton, E. R. Hedrick, P. G. Hoel, G. H. Hunt, Glenn James, J. P. LaSalle, D. H. Lehmer, S. H. Levy, Hans Lewy, G. F. McEwen, Rhoda Manning, W. A. Manning, A. D. Michal, Jerzy Neyman, W. G. Palm, W. T. Puckett, V. V. Quilliam, P. V. Reichelderfer, A. C. Schaeffer, G. E. F. Sherwood, D. V. Steed, P. M. Swingle, Gabor Szegö, A. E. Taylor, B. P. Taylor, T. Y. Thomas, S. E. Urner, F. A. Valentine, Morgan Ward, D. E. Whelan, W. F. Whitmore, W. M. Whyburn, J. R. Woolson, E. R. Worthington, Max Zorn.

A general session was held in the morning. The meeting was called to order at 9:30 A.M., and was presided over at different times by Professor G. C. Evans and Provost E. R. Hedrick. By invitation of the Program Committee, Dr. A. C. Schaeffer of Stanford University delivered an hour address on *Inequalities of A. Markoff and S. Bernstein for polynomials and related functions.*

Professor Gabor Szegö announced that Stanford University would celebrate its fiftieth anniversary next year and proposed to hold symposia in mathematics (topology, probability, and analysis) at that time.

In the afternoon a symposium on aeronautics was held under the direction of Professor Harry Bateman of the California Institute of Technology, who also presided. As Dr. Stewart was unable to be present, the chairman read the abstract of his paper and Professor von Kármán discussed it.

Following the meeting Provost and Mrs. Hedrick served tea at their home for members of the Society and their friends.

The titles and cross references to abstracts of papers read at the meeting are given below. Papers 11–17 were read in the symposium, and the symposium speakers were introduced by Professor Bateman. Abstracts whose numbers are followed by *t* were read by title.

1. H. J. Hamilton: *On monotonic and convex solutions of certain difference equations.* (Abstract 47-1-27.)

2. G. F. McEwen: *Statistical problems of the range divided by the mean in samples of size n.* (Abstract 47-1-77.)

3. J. P. LaSalle: *A note on pseudo-normed linear spaces.* (Abstract 47-1-33.)

4. D. H. Lehmer: *A general summation formula for numerical functions.* (Abstract 47-1-82.)

5. P. M. Swingle: *Indecomposable connexes.* (Abstract 47-1-109.)

6. A. E. Taylor: *Conjugations of complex Banach spaces.* (Abstract 47-1-48.)

7. W. F. Whitmore: *Convergence theorems for functions of two complex variables.* II. (Abstract 47-1-53.)

8. J. R. Woolson: *A theory of projections in complex Banach spaces.* Preliminary report. (Abstract 47-1-55.)

9. Max Zorn: *Idempotency of infinite cardinals.* (Abstract 47-1-88.)

10. Jerzy Neyman: *A statistical problem in mass production and routine analyses.* (Abstract 47-1-78.)

11. Harry Bateman: *Aerodynamical effects of changes in the fundamental equations.* (Abstract 47-1-56.)

12. J. Bjerknes: *Some uses of mathematics in meteorology.* (Abstract 47-1-58.)

13. H. J. Stewart: *Steady state oscillations in an atmosphere on a rotating sphere.* Preliminary report. (Abstract 47-1-64.)

14. Theodore von Kármán and W. R. Sears: *Solution of the problem of the airplane wing of finite span by expression in eigen-functions.* Preliminary report. (Abstract 47-1-61.)

15. M. A. Biot: *Finite difference equations applied to aircraft engine vibrations.* (Abstract 47-1-57.)

16. W. D. Rannie: *Tensor methods in the theory of turbulence.* Preliminary report. (Abstract 47-1-62.)

17. F. H. Clauser: *Exact solutions of the equations for the flow of a compressible fluid.* Preliminary report. (Abstract 47-3-149.)

18. R. M. Robinson: *On the simultaneous approximation of two real numbers.* (Abstract 47-1-84-*t*.)

19. G. A. Baker: *Maximum likelihood estimation of the ratio of the components of nonhomogeneous populations.* (Abstract 47-1-76-*t*.)

20. G. A. Baker: *Fundamental distributions of errors for agricultural field trials.* (Abstract 47-1-75-*t*.)

21. J. P. LaSalle: *Pseudo-normed linear sets over valued rings.* I. (Abstract 47-1-34-*t*.)

22. E. T. Bell: *Selective equations.* (Abstract 47-1-79-*t*.)

23. Morgan Ward: *The fundamental theorem of arithmetic.* (Abstract 47-1-12-*t*.)

T. M. PUTNAM,
Associate Secretary

THE ANNUAL MEETING OF THE SOCIETY

The forty-seventh Annual Meeting of the American Mathematical Society was held at Louisiana State University, Baton Rouge, Louisiana, from Monday, December 30, 1940 to Wednesday, January 1, 1941, in conjunction with meetings of the Mathematical Association of America and the National Council of Teachers of Mathematics.

There were four general and six sectional sessions of the Society at which three addresses and ninety research papers (forty in person) were given. All sessions were held in Nicholson Hall of Louisiana State University.

Arrangements for the meetings were made by a committee of which Professor S. T. Sanders was chairman and Professors W. V. Parker and N. E. Rutt were the local members. Through the generosity of Louisiana State University, rooms were available in three of the University dormitories.

The attendance was about 500, including the following one hundred ninety-three members of the Society:

Leonidas Alaoglu, A. A. Aucoin, W. L. Ayres, R. W. Babcock, I. A. Barnett, R. E. Basye, P. O. Bell, E. E. Betz, S. F. Bibb, D. H. Blackwell, H. E. Bray, R. W. Brink, J. C. Brixey, F. L. Brooks, R. H. Bruck, H. E. Buchanan, R. S. Burington, J. E. Burnam, L. E. Bush, W. E. Byrne, W. D. Cairns, C. C. Camp, M. E. Carlen, W. B. Carver, J. E. Case, C. R. Cassity, J. H. Chanler, E. W. Chittenden, M. D. Clement, L. M. Coffin, L. W. Cohen, Nancy Cole, J. B. Coleman, E. G. H. Comfort, H. H. Conwell, L. P. Copeland, W. A. Cordrey, N. A. Court, W. H. H. Cowles, M. J. Cox, G. F. Cramer, D. R. Curtiss, J. H. Curtiss, R. D. Daugherty, W. M. Davis, C. C. Dearman, L. J. Deck, Douglas Derry, L. L. Dines, J. L. Dorroh, W. L. Duren, John Dyer-Bennet, E. D. Eaves, Samuel Eilenberg, E. R. Elliott, H. J. Ettlinger, G. C. Evans, W. K. Feller, B. F. Finkel, H. T. Fleddermann, T. C. Fry, H. K. Fulmer, R. E. Gaskell, B. E. Gatewood, F. C. Gentry, J. S. Gold, Cornelius Gouwens, C. H. Graves, J. W. Green, R. E. Greenwood, D. W. Hall, O. H. Hamilton, E. H. Hanson, O. G. Harrold, W. L. Hart, W. W. Hart, M. C. Hartley, J. O. Hassler, K. E. Hazard, L. B. Hedge, G. A. Hedlund, R. A. Hefner, M. L. Hill, I. M. Hostetter, M. G. Humphreys, Witold Hurewicz, W. A. Hurwitz, Dunham Jackson, R. L. Jackson, Nathan Jacobson, Fritz John, F. B. Jones, M. L. Kales, H. S. Kaltenborn, D. E. Kearney, J. L. Kelley, A. J. Kempner, E. C. Kennedy, E. S. Kennedy, G. S. Ketchum, P. W. Ketchum, J. R. Kline, E. C. Klipple, J. C. Knipp, C. C. Krieger, M. E. Ladue, J. P. LaSalle, C. G. Latimer, Walter Leighton, R. G. Lubben, Dorothy McCoy, Dora McFarland, Roy MacKay, Saunders MacLane, A. E. Many, William Marshall, L. E. Mehlenbacher, A. N. Milgram, D. D. Miller, W. L. Miser, Virginia Modesitt, Deane Montgomery, Richard Morris, T. A. Mossman, E. J. Moulton, S. B. Myers, J. P. Nash, Albert Neuhaus, C. V. Newsom, E. N. Nilson, N. M. Oboukhoff, Rufus Oldenburger, E. G. Olds, E. R. Ott, F. W. Owens, H. B. Owens, W. V. Parker, E. W. Pehrson, I. E. Perlin, O. J. Peterson, B. J. Pettis, G. B. Price, A. S. Rayl, Maxwell Reade, D. P. Richardson, R. G. D. Richardson, F. D. Rigby, R. F. Rinehart, C. V. Robinson, J. B. Rosser, N. E. Rutt, S. T. Sanders, Samuel Saslaw, G. E.

Schweigert, J. A. G. Shirk, M. M. Slotnick, C. D. Smith, C. E. Smith, E. R. Smith, H. L. Smith, H. W. Smith, W. F. Smith, W. S. Snyder, R. H. Sorgenfrey, C. E. Springer, D. W. Starr, N. E. Steenrod, R. C. Stephens, R. W. Stokes, E. B. Stouffer, W. T. Stratton, Otto Szász, J. D. Tamarkin, J. M. Thomas, A. W. Tucker, L. I. Wade, T. L. Wade, C. C. Wagner, R. W. Wagner, R. K. Wakerling, R. J. Walker, J. A. Ward, L. E. Ward, W. G. Warnock, Marjorie Watson, M. J. Weiss, F. P. Welch, M. E. Wescott, R. L. Wilder, F. B. Wiley, R. M. Winger, F. L. Wren, Leo Zippin.

The meetings of the Society opened Monday afternoon with two sections, Analysis, and Algebra and Topology, with Professors G. B. Price and L. W. Cohen presiding. This was followed by a general session at which Dr. Leo Zippin gave an address entitled *Topology of rigid motions*. Professor J. R. Kline presided.

On Monday evening there were two sections, Algebra with Professor Nathan Jacobson presiding, and Geometry with Professor S. B. Myers presiding.

On Tuesday morning there was a general session for short papers at which President G. C. Evans presided. Following this Professor Saunders MacLane gave an address entitled *Extensions of groups* at a general session at which Professor J. D. Tamarkin presided.

The annual business meeting and election of officers was held on Wednesday morning. President G. C. Evans presided. This was followed by a general session at which Professor G. C. Evans gave his retiring address as President of the Society. The title of this address was *Surfaces of minimum capacity*. Vice President T. C. Fry presided at this address.

The sessions of the Society closed on Wednesday afternoon with two sections, Analysis and Topology, with Professors H. J. Ettlinger and N. E. Steenrod presiding.

Sessions of the Mathematical Association of America were held on Wednesday morning, jointly with the National Council of Teachers of Mathematics, and on Thursday morning. The National Council of Teachers of Mathematics held sessions on Monday morning, afternoon and evening, Tuesday morning, and Wednesday morning.

On Monday afternoon a tea was given in the Faculty Club by President and Mrs. P. M. Hebert of Louisiana State University.

On Tuesday afternoon there was an excursion by automobile to visit some fine antebellum homes and gardens near St. Francisville.

A joint dinner for members of all three mathematical organizations and their guests was held on Wednesday evening in Foster Hall. The attendance was two hundred ninety-one. Professor F. A. Rickey was toastmaster. He introduced Dr. P. M. Hebert, President of Louisiana State University, who welcomed the mathematicians on behalf of the

University. Other speakers, representing the three organizations, were Professors H. E. Buchanan and C. V. Newsom, and Miss Mary A. Potter. Between the speeches Miss Marguerite Muncelle entertained with music on the harp and Miss Weslyn Stephens with vocal solos. President G. C. Evans presented to Dean R. G. D. Richardson, on behalf of the Council of the Society, a beautiful hand-illuminated copy of the resolution of appreciation adopted by the Council on the occasion of his retirement as Secretary of the Society, together with a silver coffee set and for Mrs. Richardson a hand bag. President Evans also read telegrams of congratulations to Dean Richardson from the Department of Mathematics of the University of Wisconsin and from Professor Marston Morse, the newly elected President of the Society. At the close of the dinner Professor R. M. Winger presented resolutions, which were adopted unanimously, thanking Louisiana State University and the members of its Department of Mathematics for their excellent arrangements and gracious hospitality.

At the meeting of the Board of Trustees at 6:00 P.M., December 30, 1940, in the library of the Faculty Club of Louisiana State University, there was no quorum present. An adjourned meeting, therefore, was held on January 6, 1941, at 5:00 P.M. in the Men's Faculty Club of Columbia University, New York City.

The Council met on Tuesday, December 31, at 7:30 P.M. in the Faculty Club at Louisiana State University; an adjourned meeting was held at 4:15 P.M. on Wednesday at the same place.

The Secretary announced the election of the following eighteen persons to membership in the Society:

Dr. Alfredo Baños, Jr., Institute of Physics, National University of Mexico;
Mr. Virgil P. Barta, Portland, Ore.;
Professor Clair J. Blackall, College of St. Thomas, St. Paul, Minn.;
Mr. Aaron Herschfeld, U. S. Social Security Board, Washington, D. C.;
Mr. Gerhard Paul Hochschild, Princeton University;
Dr. Houston T. Karnes, Louisiana State University;
Mr. Jacob Louis Miller, Brooklyn, N. Y.;
Mr. Vernon V. Quilliam, Adohr Milk Farms, Los Angeles, Calif.;
Mr. John S. Ripandelli, Forest Hills, Long Island, N. Y.;
Dr. Raphaël Salem, Montreal, Quebec, Canada;
Professor Conway C. Sams, Mars Hill College, Mars Hill, N. C.;
Dr. Kenneth Charles Schraut, University of Dayton;
Professor Sire, University of Lyon, Lyon, France;
Professor Frederick Franklin Stephan, Cornell University;
Dr. Alfred Tarski, Harvard University;
Sister Mary Felice Vaudreuil, Mount Mary College, Milwaukee, Wis.;
Brother Bernard Alfred Welch, Manhattan College, New York, N. Y.;
Professor Antoni Zygmund, Mount Holyoke College.

It was reported that the following had been elected as nominees on the Institutional Memberships of the institutions indicated:

Brooklyn College: Mr. Bernard Greenspan.
Brown University: Dr. George Polya.
University of California: Miss Julia Hall Bowman, Messrs. George Bernard Dantzig, Stanley Phillips Frankel, Robert J. Levit, James Vernon Lewis, Martin Lewis Nelson, and Samuel A. Schaaf.
University of California at Los Angeles: Messrs. Clarence Maurice Ablow and Robert Simple Arthur.
University of Cincinnati: Mr. Harry Sears Kieval.
College of the City of New York: Mr. Joshua Barlaz.
Columbia University: Messrs. Lawrence Annenberg and Richard M. Cohen, Miss Martha Isabel Fields, Messrs. Abraham Hillman and Chung-Tsi Hsu.
Cornell University: Messrs. Norman Gustav Gunderson and Theodore Hailperin.
Harvard University: Professor Percy Williams Bridgman, Messrs. Paul Ernest Guenther and Irving Kaplansky, Professor Edwin Crawford Kemble, Messrs. George Whitelaw Mackey and Frank Moore Stewart.
University of Illinois: Messrs. Thad Paul Alton, William Franklin Atchison, Ernest Evred Blanche, Dickran Hagop Erkiletian, Arthur Edward Hallerberg, and Richard W. Hamming, Miss Mary Jeannette Hurst, Mr. Paul Ben Johnson, Miss Lois Kiefer, Messrs. Ernest Reuben Kolsrud, Brooks Javins Lockhart, Charles Willard Mathews, Gottfried Emanuel Noether, and James Ferris Price.
Institute for Advanced Study: Professor Shizuo Kakutani.
Iowa State College: Mr. Charles J. Thorne.
State University of Iowa: Mr. Franklin E. Satterthwaite.
The Johns Hopkins University: Messrs. Clagett Bowie, Carl Henry Bradshaw, and Earl Alexander Coddington, Miss Mary Theresa Curran, Dr. Frederick Weiss Light, Jr., Mr. Bernard Sherman.
University of Kentucky: Mrs. Aughtum S. Howard.
Massachusetts Institute of Technology: Messrs. Kenneth James Arnold, Elliott Lyndon Buell, and Henry Earl Singleton.
University of Michigan: Messrs. Ivor Schilansky and Bernard Vinograde.
Michigan State College of Agriculture and Applied Science: Mr. Joseph Willard Sheedy.
Northwestern University: Messrs. Arthur Saastad and Zenon Szatrowski.
University of Pennsylvania: Messrs. Nathan Jacob Fine and Joseph Lehner.
Pennsylvania State College: Dr. Allen Byron Cunningham.
The College of St. Thomas: Professor Rolf Landshoff.
Smith College: Miss Alexandra Illmer.
University of Toronto: Messrs. Nathan Saul Mendelsohn and Albert Wallace Walker.
University of Virginia: Messrs. Truman Arthur Botts and Walter Helbig Gottschalk.
University of Washington: Mr. Fumio Yagi.
University of Wisconsin: Messrs. John Oswald Danielson, Robert Stewart Gardner, Allison Potter Loomer, Nels David Nelson, Thomas Edmond Oberbeck, Lowell J. Paige, and Robert L. Wilson.
Yale University: Messrs. Andrew Mattei Gleason, Gerald Robinson MacLane, and Sidney Trachtenberg.

The following appointments by President G. C. Evans or the Coun-

cil were reported: as Committee on Arrangements for the Spring Meeting in Washington, D. C., May 2–3, 1941, Professors F. E. Johnston (chairman), E. J. Finan, T. R. Hollcroft, Florence M. Mears, J. H. Taylor, E. W. Titt, and F. M. Weida; as Committee on Arrangements for the Annual Meeting of 1941 at Lehigh University, Dean Tomlinson Fort (chairman), Professors T. R. Hollcroft, Everett Pitcher, G. E. Raynor, and W. M. Smith; as representative of the Society at the inauguration of Samuel Nowell Stevens as sixth president of Grinnell College (Grinnell, Iowa) on October 25, Professor W. J. Rusk; as representative of the Society at the inauguration of Raymon M. Kistler as president of Beaver College (Glenside, Pa.) on November 8, Professor Anna Pell Wheeler; as representative of the Society at the inauguration of Carl Stephens Ell as president of Northeastern University (Boston, Mass.) on November 19, Professor E. V. Huntington; as representative of the Society at the inauguration of James Franklin Findlay as president of Drury College (Springfield, Mo.) on November 29, Dr. F. T. H'Doubler; as an additional member of the Committee on Publicity, Professor R. J. Walker; as auditors of the Society's books for 1940, Professors R. G. Archibald and A. E. Meder, Jr.; as tellers for the election at the 1940 Annual Meeting, Dr. E. R. Lorch and Professor W. V. Parker; as new members of the Committees to Select Hour Speakers: for Annual and Summer Meetings, Professor M. H. Ingraham (committee now consists of Professors J. R. Kline, chairman, Einar Hille, C. R. Adams, and M. H. Ingraham); for Western Sectional Meetings, Professor W. J. Trjitzinsky (committee now consists of Professors W. L. Ayres, chairman, E. W. Chittenden, and W. J. Trjitzinsky); for Far Western Sectional Meetings, Professor T. Y. Thomas (committee now consists of Professors T. M. Putnam, chairman, Gabor Szegö, and T. Y. Thomas); as Colloquium speaker for 1943, Professor E. J. McShane; as representatives on the Editorial Board of the Duke Mathematical Journal for 1941–1943, Professors F. J. Murray and Morgan Ward.

The Secretary reported that the ordinary membership in the Society is now 2,336, including 213 nominees of institutional members and 78 life members. There are also 87 institutional members. The total attendance of members at all meetings in 1940 was 1,441; the number of papers read was 498; the number of invited addresses was 13; the number of members attending at least one meeting was 921.

At the annual election which closed on January 1, and at which 384 votes were cast, the following officers were elected:

President, Professor Marston Morse.

Vice President, Professor T. Y. Thomas.

Secretary, Professor J. R. Kline.

Associate Secretary, Professor T. R. Hollcroft.

Treasurer, Professor B. P. Gill.

Member of the Editorial Committee of the Bulletin, Professor L. M. Graves.

Member of the Editorial Committee of the Transactions, Professor Einar Hille.

Member of the Editorial Committee of the American Journal of Mathematics, Professor Hermann Weyl.

Member of the Editorial Committee of the Colloquium Publications, Professor G. T. Whyburn.

Members of the Council at Large, Professors L. R. Ford, G. B. Price, Tibor Radó, A. W. Tucker, and Morgan Ward.

Members of the Board of Trustees, Professors W. R. Longley, Marston Morse, and G. W. Mullins, Dean R. G. D. Richardson, and Dr. Warren Weaver.

At the business meeting following the annual election of officers, the following resolution, adopted by the Council at the Summer Meeting at Hanover, was read, and by a unanimous rising vote, the Society added its approval of this Council action:

To Roland George Dwight Richardson

The American Mathematical Society has been exceedingly fortunate in having had three devoted and able secretaries. Thomas Scott Fiske (1888–1895), Frank Nelson Cole (1895–1920), and Roland George Dwight Richardson (1921–1940). The achievements of the first two have been noted elsewhere in the archives of the Society. It is desire of the Council of the Society at its present meeting to record its grateful appreciation of the services of Richardson who will retire from office in December 1940. He was elected to the secretaryship in 1921, in the period of danger and uncertainty following the world war of 1914–1918. It was a time in the history of the Society when the never-ending tension between too slowly increasing financial resources and the costs of continuously increasing facilities for encouragement of mathematical research was most acute. By his persistent continuation of the campaigns for membership, his watchful interest in the resources of the Society, his sympathetic understanding of our research needs, and his constant and unifyingly friendly influence in the management of our affairs, Richardson has taken a most prominent part in insuring the financial safety and developing the scientific influence of the Society. He has devoted himself,

often to the limit of his strength, to our interests. In contrast to these self-sacrificing services our comments here must inevitably seem inadequate and incomplete. Richardson's real memorial lies, however, in the affection and admiration for him which are widespread among the present members of the Society, and universal among those who have known him personally and worked with him. For future generations of members the Society itself and its scientific resources will constitute an ineffaceable reminder of his effective interest.

In view of these invaluable services the Council hereby directs that a copy of these statements shall be spread upon the records of the Society, and that a copy shall also be sent, as an expression of our high regard, to our retiring Secretary.

The reports of the Treasurer and Auditors showed a balance of $6,966.39 in the general account and exclusive of the balances in the Colloquium, Sinking Fund, and other special funds. The Society's investments including Endowment Fund, Mathematical Reviews Fund, Colloquium Fund, and other special funds had a market value on November 30, 1940 of approximately $170,000. The interest income for the year was $6,909.41; dues from Institutional Memberships amounted to $6,705.00; dues from Contributing Memberships $1,114.92; dues from Ordinary Memberships $14,380.96. The Colloquium Fund showed a balance on hand of $9,718.95. The total received from the sale of the Society's publications was $18,148.02. The regular account of Mathematical Reviews showed income from subscriptions of $7,849.48, from investments of $2,469.99, and disbursements of approximately $11,800.00. The Board of Trustees adopted a budget for 1941 showing authorized expenditures and estimated receipts of the Society as $50,700 and $41,900 respectively. A sum of $1,000 from the income of the Marion Reilly Fund was appropriated for Mathematical Reviews and a similar sum for a volume of the Colloquium Publications. A copy of the Treasurer's report will, on request, be sent to any member of the Society.

On recommendation of the Committee on Places of Meetings (Professor E. W. Chittenden, chairman), the Annual Meeting of 1941 was set at Lehigh University, Bethlehem, Pa., December 30–January 1, and the Annual Meeting of 1942 for New York City in connection with the sessions of the American Association for the Advancement of Science. Times and places of meetings during 1941 were also set as follows: fall midwestern meeting at Kansas State College, Manhattan, on November 21–22, and fall far western meeting at Pasadena on November 22.

The Librarian reported that the Library of the Society now contains about 9721 volumes.

The American Journal of Mathematics, which is published jointly by the Johns Hopkins University and the Society, and to which the Society contributes an annual subvention of $2,500, printed 912 pages during 1940.

The Committee on War Preparedness presented a report. A copy may be obtained on request to Secretary Kline. The present membership of the Committee is as follows:

Chairman, Professor Marston Morse.

Subcommittee on Research, Professors Dunham Jackson (chairman), E. J. McShane, M. H. Stone, J. H. Van Vleck, Norbert Wiener, S. S. Wilks.

Subcommittee on Preparation for Research, Professors M. H. Stone (chairman), B. O. Koopman, R. E. Langer, Hans Lewy, F. D. Murnaghan, H. P. Robertson.

Subcommitee on Education for Service, Professors W. L. Hart (chairman), R. S. Burington, J. L. Coolidge, H. B. Curry, E. C. Goldsworthy, F. L. Griffin, M. H. Ingraham, E. J. Moulton.

Consultants, Aeronautics—Professor Harry Bateman, Chief; Ballistics—Professor John von Neumann, Chief, Professor W. T. Reid; Computation—Professor Norbert Wiener, Chief; Cryptanalysis—Professor H. T. Engstrom, Chief, A. A. Albert, W. A. Hurwitz, Solomon Kullback, Oystein Ore; Industry—Dr. T. C. Fry, Chief; Probability and Statistics—Professor S. S. Wilks, Chief.

The Secretary reported that during his term of office the membership of the Society had more than tripled (733–2,336).

President Marston Morse appointed as the Committee on Nomination of Officers and Members of the Council for 1942, Professors G. A. Bliss (chairman), G. C. Evans, Philip Franklin, T. H. Hildebrandt, and P. A. Smith.

The Committee on Scientific Aids to Learning with a view to aiding the use of microfilm as well as to supporting Mathematical Reviews has appropriated $10,000 for purchase of Student Microfilm Readers (see this Bulletin, p. 1). It was reported that over 6,000 notices regarding the Student Microfilm Readers had been sent to individuals and libraries and that applications for more than 350 of these machines had been received by Mathematical Reviews during the last ten days of 1940.

Revision of the By-Laws to take care of the Society's arrangements for Mathematical Reviews, to reduce the number of ex-officio voting members of the Council, and to make other minor changes, was rec-

ommended for favorable action; the revised By-Laws are to be presented for adoption at the February meeting of the Society.

A committee consisting of Professors E. W. Chittenden (chairman), T. H. Hildebrandt, H. P. Robertson, J. D. Tamarkin, and J. L. Walsh was appointed to consider and to make recommendations to the Council concerning the possible introduction of a new series of mathematical books.

The Committee on Addresses in Applied Mathematics reported through its chairman, Professor Richard Courant, recommendations for a vigorous program in connection with various meetings of mathematicians.

Titles and cross references to the abstracts of papers read at the regular sessions follow below. The papers were read as follows: Papers numbered 1–6 in the section for Analysis on Monday afternoon; papers 7–11 in the section for Algebra and Topology on Monday afternoon; papers 12–17 in the section for Algebra on Monday evening; papers 18–23 in the section for Geometry on Monday evening; papers 24–29 in the general session on Tuesday morning; papers 30–34 in the section for Analysis on Wednesday afternoon; papers 35–40 in the section for Topology on Wednesday afternoon; and papers 41–90, whose abstract numbers are followed by the letter *t*, were read by title. Mr. R. L. Swain and Miss Harlan C. Miller were introduced by Professor R. L. Moore, Mr. Nelson Robinson by Professor W. V. Parker, Professor Szolem Mandelbrojt by Professor H. E. Bray, and Dr. Russell Cowan by Professor W. G. Warnock. Paper 2 was presented by Mr. E. N. Nelson, and paper 18 by Dr. C. V. Robinson.

1. J. W. Green: *A special type of conformal map.* (Abstract 47-1-25.)

2. J. L. Walsh and E. N. Nilson: *Approximation to an analytic function by functions analytic and bounded in a region.* (Abstract 47-1-52.)

3. W. S. Snyder: *On independence of the path for line integrals of continuous functions.* (Abstract 47-1-45.)

4. J. P. LaSalle: *Pseudo-normed linear sets over valued rings.* II. (Abstract 47-1-95.)

5. I. E. Perlin: *Indefinitely differentiable functions of several real variables.* (Abstract 47-1-40.)

6. L. B. Hedge: *Transformations of multiple Fourier series.* Preliminary report. (Abstract 47-1-28.)

7. N. E. Rutt: *Rectangular arrays of combinations.* (Abstract 47-1-8.)

8. R. W. Wagner: *The differentials of analytic matrix functions.* (Abstract 47-1-11.)

9. R. H. Sorgenfrey: *Concerning triodic continua.* (Abstract 47-1-105.)

10. R. L. Swain: *Distance axioms in Moore spaces.* (Abstract 47-1-106.)

11. J. L. Kelley: *A theorem on transformations.* (Abstract 47-3-178.)

12. T. L. Wade: *Tensor algebra and Young's symmetry operators.* (Abstract 47-1-10.)

13. C. C. Camp: *A root cubing method of solving equations.* (Abstract 47-1-2.)

14. L. I. Wade: *Certain quantities transcendental over the field* $GF(p^n, x)$. (Abstract 47-1-87.)

15. Douglas Derry: *On the boundary case of Minkowski's linear form theorem.* Preliminary report. (Abstract 47-1-81.)

16. Rufus Oldenburger: *The minimal number problem.* (Abstract 47-1-6.)

17. R. H. Bruck: *The structure of the rational representations of a wide class of linear groups.* (Abstract 47-1-1.)

18. L. M. Blumenthal and C. V. Robinson: *Helly theorems on the sphere.* (Abstract 47-1-67.)

19. N. A. Court: *On the skew cubic.* (Abstract 47-1-73.)

20. E. R. Ott: *A locus determined by an algebraic correspondence.* (Abstract 46-11-497.)

21. P. O. Bell: *On differential geometry intrinsically connected with a surface element of projective arc length.* (Abstract 47-1-65.)

22. C. R. Cassity: *The double points of a pencil of cubics invariant under the quadratic transformation.* (Abstract 47-1-68.)

23. Nelson Robinson: *On the contact of a quartic surface with a general analytic surface.* (Abstract 47-3-164.)

24. L. L. Dines: *On the mapping of quadratic forms.* (Abstract 47-1-19.)

25. J. B. Rosser: *Explicit bounds for some functions of prime numbers.* (Abstract 47-1-85.)

26. S. B. Myers: *Complete Riemannian manifolds of positive mean curvature.* (Abstract 47-1-74.)

27. J. M. Thomas: *Orderly differential systems.* (Abstract 47-1-49.)

28. A. N. Milgram: *Extensions of decompositions.* (Abstract 47-1-97.)

29. Szolem Mandelbrojt: *On a Dirichlet series.* (Abstract 47-3-137.)

30. B. E. Gatewood: *Thermal stresses in regions bounded by epitrochords.* (Abstract 47-1-59.)

31. Russell Cowan: *A method of solving a linear difference equation with polynomial coefficients of degree m.* (Abstract 47-1-16.)

32. J. H. Curtiss: *Trigonometric interpolation by means of the complex Lagrange polynomial.* (Abstract 47-1-18.)

33. Fritz John: *On the character of solutions of hyperbolic equations.* Preliminary report. (Abstract 47-1-32.)

34. Otto Szász: *On the partial sums of harmonic developments and related power series.* (Abstract 47-1-47.)

35. Harlan C. Miller: *Concerning certain types of end points of compact continua.* (Abstract 47-1-98.)

36. G. E. Schweigert: *Border transformations.* (Abstract 47-1-104.)

37. O. G. Harrold: *On a class of continuous maps.* (Abstract 47-1-92.)

38. R. L. Wilder: *Characterizations of those euclidean domains whose boundaries are lc^k.* (Abstract 47-1-112.)

39. Samuel Eilenberg: *Imbedding of spaces into euclidean spaces.* Preliminary report. (Abstract 47-1-90.)

40. F. B. Jones: *Topologically flat spaces.* (Abstract 47-1-93.)

41. S. E. Warschawski: *On conformal mapping of infinite strips.* (Abstract 47-3-146-*t*.)

42. R. P. Boas and D. V. Widder: *Completely convex functions.* Preliminary report. (Abstract 47-1-15-*t*.)

43. W. D. Duthie: *Segments in ordered sets.* (Abstract 47-1-3-*t*.)

44. P. M. Swingle: *An abstraction of types of connected sets.* (Abstract 47-1-108-*t*.)

45. L. M. Blumenthal: *Betweenness in metric ptolemaic spaces.* (Abstract 47-1-66-*t*.)

46. J. J. Eachus: *Classification of solutions and of pairs of solutions of $y'''+2p(x)y'+p'(x)y=0$ by means of boundary conditions.* (Abstract 47-1-21-*t*.)

47. C. N. Moore: *On the Cesàro and Abel-Poisson summability of the differentiated double Fourier series.* (Abstract 47-1-38-*t*.)

48. H. L. Garabedian: *Relations between hypergeometric methods of summation.* (Abstract 47-1-24-*t*.)

49. C. W. Vickery: *A new proof of a theorem of Chittenden.* (Abstract 47-1-111-*t*.)

50. J. Marcinkiewicz and Antoni Zygmund: *On the behavior of trigonometric and power series.* (Abstract 47-1-36-*t*.)

51. Hyman Serbin: *Upper bounds for the remainder of certain power series.* (Abstract 47-1-44-*t*.)

52. H. S. Wall: *A theorem on real functions bounded in the unit circle.* (Abstract 47-1-51-*t*.)

53. R. E. Greenwood: *Hankel and other extensions of Dirichlet series.* (Abstract 47-1-26-*t*.)

54. Walter Leighton: *On the convergence of continued fractions.* (Abstract 47-1-35-*t*.)

55. Warren Ambrose: *Representation of ergodic flows.* Preliminary report. (Abstract 47-1-14-*t*.)

56. A. E. Heins: *On the solution of partial difference equations. Two interval "boundary conditions."* Preliminary report. (Abstract 47-1-29-*t*.)

57. N. E. Rutt: *Sets with a certain sort of derived set.* (Abstract 47-1-103-*t*.)

58. Nathaniel Coburn: *A note on conformal geometry.* (Abstract 47-1-69-*t*.)

59. Nathaniel Coburn: *Unitary spaces with corresponding geodesics.* (Abstract 47-1-70-*t*.)

60. H. M. Schwartz: *Convex functions and the law of the mean.* (Abstract 47-1-42-*t*.)

61. H. M. Schwartz: *Sequences of Stieltjes integrals.* (Abstract 47-1-43-*t*.)

62. Arthur Sard: *The measure of the critical values of differentiable maps of euclidean spaces.* (Abstract 47-1-41-*t*.)

63. M. F. Smiley: *Measurability and distributivity in the theory of lattices.* (Abstract 47-1-9-*t*.)

64. Jesse Douglas: *Solution of the inverse problem of the calculus of variations.* (Abstract 47-1-20-*t*.)

65. J. H. Curtiss: *Degree of polynomial approximation on a lemniscate.* (Abstract 47-1-17-*t*.)

66. N. A. Court: *On the harmonic pole.* (Abstract 47-1-72-*t*.)

67. G. D. Nichols: *A sufficient condition for Cesàro summability.* (Abstract 47-1-39-*t*.)

68. R. L. Wilder: *On the domains complementary to continua having certain avoidability properties.* (Abstract 47-1-113-*t*.)

69. R. L. Wilder: *The duality between the S-properties of closed sets and their complements in the euclidean n-sphere.* (Abstract 47-1-114-*t*.)

70. Hermann Weyl: *Theory of reduction for arithmetical equivalence.* II. (Abstract 47-1-13-*t*.)

71. C. J. Everett: *An extension theory for rings.* (Abstract 47-1-4-*t*.)

72. D. V. Widder: *Necessary and sufficient conditions for the representation of a function in Lidstone series.* (Abstract 47-1-54-*t*.)

73. Eric Reissner: *A new derivation of the equations for the deformation of elastic shells.* (Abstract 47-1-63-*t*.)

74. Harlan C. Miller: *On the existence of a certain type of irreducible continuum.* (Abstract 47-1-99-*t*.)

75. Harlan C. Miller and R. L. Swain: *Properties of two intersecting arcs.* (Abstract 47-1-100-*t*.)

76. R. L. Swain: *Linear metric space.* (Abstract 47-1-107-*t*.)

77. Abraham Spitzbart: *Approximation in the sense of least pth powers by polynomials with a single auxiliary condition of interpolation.* (Abstract 47-1-46-*t*.)

78. F. B. Hildebrand: *The approximate solution of singular integral equations arising in engineering practice.* (Abstract 47-1-60-*t*.)

79. Leonard Carlitz: *Finite differences and polynomials in a Galois field.* (Abstract 47-1-80-*t*.)

80. Samuel Eilenberg: *Monotone families of manifolds.* (Abstract 47-1-91-*t*.)

81. S. B. Myers: *Compact groups whose elements are of finite order.* (Abstract 47-1-101-*t*.)

82. M. R. Hestenes: *Extension of the range of a differentiable function.* (Abstract 47-1-30-*t*.)

83. Richard Courant: *Critical points and unstable minimal surfaces.* (Abstract 47-1-71-*t*.)

84. W. J. Trjitzinsky: *Analytic theory of singular elliptic partial differential equations.* (Abstract 47-1-50-*t*.)

85. Gordon Pall: *Simultaneous representation in a quadratic and linear form.* (Abstract 47-1-83-*t*.)

86. N. M. Oboukhoff: *On the validity of total differential as the principal part of the increment of a function of two or more variables.* (Abstract 47-3-138-*t*.)

87. Oscar Zariski: *Pencils on an algebraic variety and a new proof of a theorem of Bertini.* (Abstract 47-3-167-*t*.)

88. Edward Kasner and J. J. DeCicco: *The classification of analytic arcs or elements under the group of arbitrary point transformations.* (Abstract 47-3-162-*t*.)

89. R. P. Agnew: *On methods of summability and mass functions determined by hypergeometric coefficients.* (Abstract 47-3-125-*t*.)

90. E. S. Allen and Harvey Diehl: *The enumeration of glycols.* (Abstract 47-3-147-*t*.)

W. L. AYRES,
Associate Secretary

ROBERT EDOUARD MORITZ—IN MEMORIAM

The sudden death of Professor Moritz in Seattle on December 28, 1940, not only marked the close of a life that had measured the allotted span, but terminated an honorable career of nearly half a century in the service of mathematics and higher education. Retired to half time two years before, he had taught through the fall quarter and had made plans to spend the winter in California. He occupied the position of head of the department of mathematics at the University of Washington for over thirty years, at various periods of which astronomy was included. He witnessed the growth of the university population from a total of some eight hundred for the academic year to approximately eleven thousand, including one thousand graduate students, in a single quarter. It was largely due to his high standards, energy and vision that the department of mathematics was competently manned and adequately equipped with library and other facilities to keep pace with this development.

Robert Edouard Moritz was born at Christianstahl in Schleswig-Holstein, the son of Karl R. and Maria Stahlhut Moritz on June 2, 1868. His early education was received in the public schools of Germany. At the age of twelve he came to the United States with his parents who settled on a farm in Nebraska. On the casual advice of a friend, young Moritz entered Hastings Academy in 1885, expecting to spend only a single term. But one term sealed his fate. By dint of the utmost frugality, of doing his own cooking, of sharply curtailing the normal hours of sleep, he was able to eke out his meagre earnings to cover the bare necessities and to remain in school—remain for seven years, at the end of which he was graduated from Hastings College in 1892. "He worked his way through college" has become a commonplace in American biography. It sounds like a simple formula, and there are of course many versions. But anyone lacking the actual experience can have only a faint conception of what those few words might connote in the short-grass region of Nebraska in the late eighties and early nineties—those years of acute and prolonged distress in the prairie states, under the scourge of hot winds, crop failures and mortgage foreclosures.

After a year spent in graduate study at the University of Chicago he returned to his Alma Mater where he held the chair of mathematics and physics from 1893 to 1898. In the meantime he attended the University of Chicago during two summer quarters, completing the requirements for the master's degree in 1896. That same summer he married Miss Cassia L. Kennedy of Aurora, Nebraska, who with two

sons survive him—one, Harold K. Moritz is associate professor of civil engineering at the University of Washington, the other, Donald B. Moritz lives in Minneapolis. An appointment as instructor in mathematics enabled him to continue his graduate work at the University of Nebraska, which awarded him the Ph.D. in 1901. The next year he studied abroad, chiefly under H. Weber and Th. Reye at the University of Strassburg, earning a second Ph.D. in 1902. Returning to the University of Nebraska, he served as adjunct professor for one year and as assistant professor for another.

In 1904 he was appointed professor and head of the Department of Mathematics and Astronomy at the University of Washington, succeeding the late Arthur Ranum. He threw himself at once into the scientific and educational work of the community and state. One of his first duties was to repair the university telescope, the lenses having been stolen the preceding summer. He prepared for a Seattle Sunday paper an article with diagrams on the forthcoming solar eclipse. And he opened the observatory to the public on certain nights, a custom that has prevailed to the present time. Holding a profound conviction of the value of mathematics in the curriculum, he wrote numerous articles for educational journals, both state and national. He was a principal speaker at the three-day sessions of one of the county institutes, discussing various aspects of the teaching of elementary mathematics on several of the programs. His final address seems to have made a deep impression for it was printed in full by popular demand in the local weekly paper.

The bibliography of his published books and papers contains about 75 titles, including his educational papers which comprise about a third of the total. In addition, he was a clever problem solver and for a dozen years beginning in 1920 he was a regular contributor to the problem department of the American Mathematical Monthly. His mathematical articles range over the fields of analysis, number theory, geometry and economics. Perhaps the most noteworthy are: *Generalization of the differentiation process* [Quotientiation], American Journal, 1902; *Ueber gewisse Infinitesimal-Operationen der hoeheren Operations-Stufen* (three parts totaling 60 pages), Tôhoku Mathematical Journal, 1921, 1922, 1923; and the monograph on *Cyclic-Harmonic Curves*, University of Washington Publications in Mathematics, 1923, pp. 1–58 plus 21 plates. The last is a discussion of the curves represented by the polar equation $\rho = a \cos (p/q)\theta + k$, where p and q are integers. A machine for drawing these curves, when $p, q = 1, 2, \cdots, 10$, built according to his design in the University shops, is in the possession of the Department of Mathematics.

His book on *Plane and Spherical Trigonometry* (John Wiley and Sons, 1911, 1913) is one of the most comprehensive treatises on trigonometry ever published in America. His *Short Course in Mathematics* (Macmillan, 1919) was an outgrowth of the course prescribed for the Students' Army Training Corps in the fall of 1918 when the University was turned into a training camp for soldiers, sailors and marines. Perhaps the best known is his *Memorabilia Mathematica* (Macmillan, 1914) the results of the painstaking labor of ten years, which is read and valued by every lover of mathematics.

He was an expert teacher, especially for the stronger students, whom he stimulated to their best efforts. Many factors unite to make the great teacher but a major secret of his success was his manifest sincerity and perennial enthusiasm.

He, usually accompanied by Mrs. Moritz, traveled widely—went several times to Europe, twice to the Orient, once around the world. They visited Alaska and Mexico and explored every section of the United States in their Model T.

He was a member of the American Mathematical Society since 1904 and a charter member of the Mathematical Association of America, serving on the first board of trustees, 1916–1918. He maintained his mathematical interest and activity to the last, reading a paper at the recent international congress at Oslo. His latest paper was presented before the departmental Research and Journal Club last spring on the centroid of a simplex in n-dimensional space.

R. M. WINGER

BOOK REVIEWS

Stencils for Solving $x^2 \equiv a \pmod{m}$. By Raphael M. Robinson. Berkeley and Los Angeles, University of California Press, 1940. 14 pp., 272 cards.

These stencils enable one to find directly all the solutions of the given congruence for $m \leqq 3000$. The range of m may be extended by additional computation.

In brief, the theory back of the construction is as follows: to solve the congruence we must find a y such that $a+my$ is a quadratic residue of all numbers E; that is, if m is prime to E, $a/m+y$ and m must have the same quadratic character (mod E) when the quadratic character for $E=16$ is the value of m (mod 8). It is not hard to see that we need consider only $y \leqq m/4$. Each card contains the numbers from 1 to 750, it is headed with a value of E (9, 16 or one of the seven primes from 5 to 23 inclusive), a value of a/m (mod E) and a quadratic character of m (mod E). Holes are punched in the values of y for which $a/m+y$ has the same quadratic character as m (mod E). Then to solve $x^2 \equiv a \pmod{m}$ one selects the cards appropriate to a and m for the nine values of E, stacks them and looks through the holes.

The stencils are very simple in principle and convert drudgery into fun.

BURTON W. JONES

The Consistency of the Axiom of Choice and of the Generalized Continuum-Hypothesis with the Axioms of Set Theory. By Kurt Gödel. (Annals of Mathematics Studies, no. 3.) Princeton, University Press; London, Humphrey Milford and Oxford University Press, 1940. 66 pp. $1.25.

Abstract. In this study it is proved that the axiom of choice and Cantor's generalized continuum-hypothesis are consistent with the other axioms of set theory if these latter axioms are themselves consistent. The system Σ of axioms here adopted for set theory is essentially that of P. Bernays (Journal of Symbolic Logic, vol. 2, p. 65).

In Σ the primitive notions are: *class*, *set*, and the relation ϵ between class and class, class and set, set and class, or set and set. The first two axioms of Σ specify the relation between classes and sets: Axiom A1. Every set is a class. Axiom A2. Every class, which is a member of some class, is a set. As a consequence of this distinction between class and set, there exists a *universal class* V of all sets in the system Σ. The remaining axioms of Σ are of a form conventional in ordinary

set theory, and deal with intersection, complement, power set, etc.

Chapter I of this study includes a statement of the axioms of Σ together with a few preliminary notions, such as *ordered pair* and *single-valued class* of ordered pairs. Chapter II is concerned with the following topics: extension of propositional functions, definition and properties of relations and functions, and properties of operations with classes and sets. Chapter III is devoted to an exposition of ordinal numbers (based on the work of J. von Neumann) in which an ordinal α is the class of all ordinals less than α; for example, $0 =$ the null set, $1 = \{0\}$, $2 = \{0, 1\}$, etc. Chapter IV develops the theory of cardinal numbers. In Chapters V and VI a model Δ is constructed for the system Σ on the assumption that Σ is consistent, for the classes and sets of Δ are merely those classes and sets of Σ characterized as *constructible* (the definition of constructibility being based on certain properties of ordinal numbers). Let L denote the class of all constructible sets in the universal class V. In Chapter VII it is proved that, when the axioms of Σ are applied just to the model Δ, V being the class of all sets in Δ and L being the class of sets constructible within Δ, then $V = L$, i.e., every set in Δ is constructible. Thus, if there exists a model for the system Σ, then there also exists a model for the system Σ augmented by the axiom $V = L$, this latter model consisting of the classes and sets constructible within the given model for Σ. The existence of this latter model shows that, if the system Σ is consistent, then the augmented system Σ' is consistent, where Σ' comprises the axioms of Σ together with the axiom $V = L$. In the last chapter it is proved that the axiom of choice and Cantor's generalized continuum hypothesis are deducible as theorems in the system Σ'.

While a number of subjects are discussed in this study, the material actually developed is largely confined to just those lemmas needed to attain the final result.

C. C. TORRANCE

Metric Differential Geometry of Curves and Surfaces. By Ernest Preston Lane. Chicago, University Press, 1940. 216+8 pp. $3.00.

This book is designed as a text for beginning graduate students; it is not intended to be an exhaustive treatise on the subject. The order and content of the material has been used by the author in his classes for several years. It has therefore been thoroughly subjected to the acid tests of the classroom for teachability and for readability. It should on that account be of much service to the classroom teacher and to the student in his independent reading.

Although the material selected is classic, the point of view in pres-

entation is definitely modern. Definitions are so given as to be valid in projective space or in hyperspace if such definitions are possible. Scattered throughout the text are some 330 exercises. Many of these exercises are mere applications of the material in the text proper, but many require developments found in the textual material in the classic treatises. Several of these exercises show the influence of projective methods and material on the author.

The subject matter is well indicated by the chapter headings: Chapter I, Curves; II, The Moving Trihedron; III, Surfaces; IV, Curves on Surfaces; V, Curvature; VI, Transformations of Surfaces. A short bibliography of the classic treatises on Metric Differential Geometry is appended.

Of especial interest is the treatment of the moving trihedron and its application to curves in one-to-one correspondence. Let a curve C' be in one-to-one correspondence with a given curve C so that P' on C' corresponds to P on C. The direction cosines of the tangent line, principal normal, and binormal of C' at P' referred to the local trihedron of C at P, are derived as functions of the local coordinates of P', the derivatives of the same, and the curvature and torsion of C at P. Applications are made to involutes, evolutes, parallel curves, transformation of Combescure, Bertrand curves. Other applications are left to the student as exercises.

In the chapter on surfaces, a lucid exposition is given of envelopes of one parameter families of surfaces. As often occurs, loci associated with space curves are simple when written in local coordinates. Methods of finding envelopes applicable in this case are given. A fresh treatment is also made of ruled surfaces other than developables.

In the chapter on curves on surfaces, the relation between orthogonality, conjugacy, and harmonic separation is stressed. In this chapter too, the Gauss and Weingarten differential equations and the Gauss-Codazzi integrability conditions are derived.

The discussion of the Gaussian and mean curvatures is well presented in Chapter V, the problems of the chapter excellently supplementing the textual material.

Conformal mapping, applicability, equiareal maps, and parallel surfaces are briefly discussed in the concluding chapter. If time permitted for such a discussion, material in this Chapter VI could no doubt have been unified in a manner similar to that used for curves in one-to-one correspondence by using the methods developed by the author in his paper *The moving trihedron*, Transactions of this Society, vol. 36 (1934), pp. 696–710.

Although some teachers may fail to find some of the familiar material, the text is ample for a first course. Moreover the exercises give much of the factual material of the missing topics.

The result (7.12) in exercise 2, p. 32 seems to be incorrect. Problem 17 on page 121 would have been more accurately expressed by speaking of an *analytic* function of a complex variable. The exercises 4 and 5 on page 140 are incorrect. The University of Chicago Press may well feel proud of this book in every respect. It is my hope that the time taken in its preparation will be repaid by its wide use. Written by a master of oral and written exposition it should receive wide acclaim.

VERNON G. GROVE

Lattice Theory. By Garrett Birkhoff. (American Mathematical Society Colloquium Publications, vol. 25.) New York, American Mathematical Society, 1940. 6+155 pp. $2.50.

This is the first book on the far-reaching subject of lattices. The author has succeeded in giving a comprehensive, yet not too terse, account of the theory of lattices and its relation to other branches of mathematics. The general plan is to devote six chapters to the abstract theory, and three to applications. Chapter I deals with partially ordered systems, Chapter II with lattices and their general properties; the important modular axiom is assumed in Chapter III, and the axiom of complementation is added in Chapter IV. Distributive lattices and Boolean algebras end the abstract theory in Chapters V and VI. Thus the author takes the reader through the most important general classes of lattices, by imposing successively restrictive conditions. The last three chapters apply lattice theory to function theory, logic and probability theory.

A partially ordered system is introduced in Chapter I as a set L with a binary relation $\geqq$ on L which is reflexive and transitive and has the property that $x \geqq y$, $y \geqq x$ implies $x = y$. Most important in connection with general partially ordered systems are the principle of duality, viz., that L together with the converse $\leqq$ of the relation $\geqq$ forms a partially ordered system, and a study of the so-called chain conditions, one or both of which are satisfied in many examples. The ascending chain condition asserts that no infinite sequence (a_i) with $a_i < a_{i+1}$ exists; the descending chain condition is dual. Many examples of a partially ordered system are cited, a few of which are the set of all subsets of a set, classes of distinguished subsets of a set, real numbers, integers (relative to the relation of divisibility), partitions, and topologies on a space.

Most of the familiar examples of a partially ordered system are lattices, i.e., there exist for any two elements x, $y \in L$ a gr.l.b. $x \cup y$ and l.u.b. $x \cap y$. Indeed, many examples form *complete* lattices, i.e., lattices in which every subset has a gr.l.b. and l.u.b. The prototype of complete lattices is the lattice of all subsets of a set having a given extensionally attainable property (E. H. Moore), i.e., all *closed* sets relative to an abstract closure operation $\overline{A}$ having the properties $\overline{A} \geqq A$, $\overline{\overline{A}} = \overline{A}$, and $\overline{A} \geqq \overline{B}$ when $A \geqq B$. The MacNeille solution to the problem of embedding a partially ordered system in a complete lattice is given, and a number of other topics are discussed, including interesting questions regarding free lattices. Finally, it is shown that there exists in a lattice an intrinsic topology defined wholly in terms of the order relation.

The modular axiom states that $a \leqq c$ implies $(a \cup b) \cap c = a \cup (b \cap c)$. Among the examples cited which satisfy the modular axiom are the set of all invariant subgroups of a group (with operators), the set of all subspaces of a linear space, the set of right (left) or two-sided ideals in a ring, and the linear subspaces of an abstract projective geometry. The main result of Chapter III is that modularity implies the Jordan chain condition, which states that two principal chains connecting the same elements of the lattice are equal in length.

Complemented modular lattices are those lattices with zero O and unit I in which each element a has at least one "complement" x such that $a \cup x = I$, $a \cap x = O$. The fundamental result in Chapter IV characterizes abstract projective geometries (in the sense of Veblen and Young) as complemented modular lattices which are *simple* (in the sense of abstract algebra). This result is due to the author. The generalization to "continuous geometries" due to von Neumann is briefly described.

Chapters V and VI contain a study of distributive lattices, i.e., those in which $(a \cup b) \cap c = (a \cap c) \cup (b \cap c)$, and boolean algebras, i.e., complemented distributive lattices. The representation theory, which connects abstract boolean algebras with algebras of sets, and the applications of boolean algebras to topology are due chiefly to Stone.

The application in Chapter VII of lattice theory to function theory is more in the nature of an abstract theory of linear spaces which are partially ordered. Applications to logic in Chapter VIII are quite natural; logics considered are Brouwerian logic, classical logic and quantum logic. The final chapter continues Chapter VIII: A probability functional on a boolean algebra is a positive functional $p(x)$ such that $p(x \cup y) + p(x \cap y) = p(x) + p(y)$, and $p(O) = 0$, $p(I) = 1$. Probability functionals are found to lie in a vector lattice; the properties of this

lattice are investigated. The chapter closes with a consideration of ergodic theorems.

Perhaps the most striking features of the book are the homogeneity of presentation and the thoroughness with which so large a subject is covered in so small a space. At the end the author includes a list of seventeen unsolved problems, which should prove provocative to the reader. The reviewer was especially pleased to find the first lattice-theoretic treatment ever given of Moore's extensional attainability, a simple but extremely useful tool. The section on polarity in Chapter II is also worthy of notice because of its wide applicability. The book is remarkably free from errors, the only ones noted being a few trivial errors in typography.

L. R. WILCOX

NOTES

On Friday, December 27, there was held at the University of Pennsylvania the meeting of Section A of the Association for the Advancement of Science. At that time Professor Marston Morse of the Institute for Advanced Study gave his address as retiring Chairman of the Section. He spoke on *A mathematical theory of equilibrium with applications to minimal surfaces*. By special invitation Dr. C. G. Rossby of the United States Weather Bureau addressed the Section on *The mathematical problems in meteorology*. Papers read before Section A at these meetings were, by special arrangement, regarded as having been read before the Society by title. In this manner papers 61, 62, 84 and 90 of the report of the annual meeting were presented. About one hundred persons were present at the meetings.

Stanford University is celebrating its fiftieth anniversary during the year 1941. In this connection a larger than usual summer quarter program is planned. Professor G. T. Whyburn from the University of Virginia will give a course on Combinatorial Topology, Professor Einar Hille from Yale University a course on Selected Topics in Theory of Functions. During the summer, presumably in the first part of August, a two-day symposium is planned with conferences on Topology, Calculus of Probability and Statistics, Theory of Functions, and general subjects. The following speakers have been secured so far: E. T. Bell, H. F. Blichfeldt, Einar Hille, A. D. Michal, Jerzy Neyman, George Polya, G. T. Whyburn, Antoni Zygmund. The program will be announced later.

The National Council of Teachers of Mathematics held its twenty-first annual meeting in Atlantic City on February 21–22, 1941. Details of the program and places of meeting are given in the February issue of The Mathematics Teacher.

A Survey of Methods of Apportionment in Congress by Professor E. V. Huntington has recently been published. Copies can be obtained from the Superintendent of Documents, Washington, D. C. for ten cents each.

A plaque has recently been presented to Washington University by the United States Naval Academy in honor of William Chauvenet who was appointed professor of mathematics and astronomy at this University in 1852 and chancellor in 1862. The plaque is a replica of one at the Academy and it bears the inscription: "William Chauvenet,

Professor of Mathematics, United States Navy and President of the Academic Board from 1847 to 1850, largely through whose efforts and plan the Naval Academy was established and organized at Annapolis."

The Sylvester Medal of the Royal Society has been awarded to Professor G. H. Hardy of the University of Cambridge for his important contributions to pure mathematics.

Professor Abraham Cohen of Johns Hopkins University has been given the title of professor emeritus.

Professor R. A. Fisher of University College, London, will be visiting professor of experimental statistics at North Carolina State College during the summer session June 16–July 25, 1941.

Dr. A. D. Bradley of Hunter College has been promoted to an assistant professorship.

Dr. A. A. F. Brown has accepted a position in the statistical department of L. Bamberger and Company in Newark, New Jersey.

Dr. G. W. Brown has taken a position in the research department of R. H. Macy and Company of New York City.

Assistant Professor Gertrude M. Cox of Iowa State College has been appointed professor of experimental statistics and director of the statistical laboratory at North Carolina State College.

Mr. R. D. Dorsett of the University of Oklahoma has been promoted to an assistant professorship.

Mr. L. T. Dunlap of Pennsylvania State College has been promoted to an assistant professorship.

Assistant Professor E. J. Finan of the Catholic University of America has been promoted to an associate professorship.

Dr. H. C. Fryer of Kansas State College has been promoted to an assistant professorship.

Associate Professor Gordon Fuller of Alabama Polytechnic Institute has been promoted to a professorship.

Dr. Beatrice L. Hagen of Pennsylvania State College has been promoted to an assistant professorship.

Assistant Professor J. J. L. Hinrichsen of Iowa State College has been promoted to an associate professorship.

Assistant Professor F. H. Hodge of Purdue University has retired.

Dr. L. P. Hutchison of Indiana Technical College has been appointed to an assistant professorship at The Citadel, Charleston, South Carolina.

Dr. Fulton Koehler of the University of Minnesota has accepted a position in the actuarial department of the Northwestern National Life Insurance Company.

Dr. Szolem Mandelbrojt, formerly of the College of France, has been appointed visiting lecturer at the Rice Institute.

Dr. G. E. Moore of the University of Illinois has been promoted to an assistant professorship.

Dr. H. R. Phalen has been appointed to an associate professorship at the College of William and Mary.

Professor H. R. Pyle of Earlham College has been appointed to a professorship at Whittier College.

Associate Professor C. B. Read of the University of Wichita has been promoted to a professorship.

Dr. S. T. Sanders, Jr., of Delta State Teachers College has been appointed to a professorship at Southwestern Louisiana Institute.

Associate Professor I. M. Sheffer of Pennsylvania State College has been promoted to a professorship.

Assistant Professor D. T. Sigley of Kansas State College has been promoted to an associate professorship.

Assistant Professor S. B. Townes of the University of Oklahoma has been promoted to an associate professorship.

Assistant Professor G. C. Vedova of St. John's College has been appointed to an assistant professorship at the University of Maryland.

Dr. Henry Wallman of the Institute for Advanced Study has been appointed to an assistant professorship at the University of North Carolina.

Associate Professor Morgan Ward of the California Institute of Technology has been promoted to a professorship.

Dr. G. C. Webber of the University of Delaware has been promoted to an assistant professorship.

Professor Oscar Zariski of Johns Hopkins University is on leave of absence for 1940–1941 and is at Harvard as a visiting lecturer.

Dr. M. M. Day of the University of Illinois has been made an associate in mathematics.

The following appointments to instructorships are announced: University of California at Los Angeles: Dr. Ralph Byrne, Jr.; Columbia University: Dr. Walter Strodt; Cornell University: Mr. N. G. Gunderson, Mr. R. R. R. Luckey; University of Dayton: Dr. K. C. Schraut; Georgia School of Technology: Mr. L. B. Williams; University of Illinois: Dr. R. S. Pate; Iowa State Teachers College: Dr. H. C. Trimble; Itasca Junior College: Dr. H. L. Olson; University of Kansas: Mr. L. R. Shobé; Northwestern University: Mr. H. E. Burns; Ohio State University: Dr. W. J. Schart; Oregon State College: Dr. P. C. Hammer; Rutgers University: Dr. J. H. Giese; University of Texas: Mr. J. R. Foote, Mr. H. J. Jones, Dr. A. M. Mood, Mr. I. C. Roberts; Trinity College, Washington, D. C.: Dr. Mary Varnhorn; Vermont Junior College: Mr. D. F. Johnson; University of Wichita: Mr. Edison Greer; University of Wisconsin: Dr. C. B. Smith; College of Wooster: Mr. M. P. Fobes.

The death on December 4, 1940 of Mr. E. H. Hayes, formerly mathematical tutor of New College, Oxford, at the age of eighty-six years has been reported.

The death of Dr. Myron Mathisson, formerly of the University of Warsaw, is reported in Nature.

Professor I. Schur of the University of Berlin died on January 10, 1941, in Tel Aviv.

Rev. Canon Alfred Young, formerly lecturer in mathematics in Selwyn College, Cambridge, died on December 19, 1940, at the age of sixty-seven years.

Dean W. C. Graustein of Harvard University died January 22, 1941. He had been a member of the Society since 1911. A more adequate notice will appear in a later issue.

Professor G. A. Knapp of Maryville College, Maryville, Tennessee, died in November, 1940.

Assistant Professor C. R. Wilson of Rutgers University died in July, 1940. He had been a member of the Society since 1929.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA

115. J. H. Bell: *Topics related to the factorization of matrices.*

This paper deals with two problems that are of particular interest in the solution of the unilateral matrix equation. Let $\mathfrak{D}$ be a quasi-field and $\mathfrak{D}[\lambda]$ the polynomial ring of $\mathfrak{D}$, where λ is a commutative indeterminate. The first problem is: Under what conditions is the matrix A with elements in $\mathfrak{D}[\lambda]$ the left associate of a matrix which is proper of degree k in λ? A necessary and sufficient condition is obtained. The proof may be shortened in the case that $\mathfrak{D}$ is a field $\mathfrak{F}$. The second problem, which is considered only in the case of a field $\mathfrak{F}$, is: Given two nonsingular matrices A and B with elements in $\mathfrak{F}[\lambda]$; if A and B are not left associates, what other proper right divisors of the least common left multiple of A and B exist which are not left associates of A or B? Various results are obtained for the second problem. These results may be used, with the result obtained in solving the first problem, to show the existence of families of solutions of the unilateral matrix equation, if two solutions satisfying certain conditions are known to exist. (Received January 23, 1941.)

116. A. H. Clifford: *Semigroups admitting relative inverses.*

A semigroup is a system S closed under an associative binary operation: $(ab)c = a(bc)$. S is said to admit relative inverses if for each a in S there exists an element e of S such that (1) $ea = ae = a$, and (2) $aa' = a'a = e$ for some a' in S. The "gross structure" of such an S is determined by showing that S is the class sum of mutually disjoint semigroups S_α of known structure (A. Suschkewitsch, D. Rees) which can be arranged in a semi-lattice P such that the set-product $S_\alpha S_\beta$ of any two of them is wholly contained in the "cross-cut" S_γ of S_α and S_β in P. If the idempotent elements of S commute with each other, then the S_α are groups, and the structure of S can be given completely. (Received January 13, 1941.)

117. A. H. Clifford and Saunders MacLane: *Factor sets of a group in its abstract unit group.*

This paper is an investigation of the structure of the set of group extensions of a certain abstract unit group. This group is one which arises in class field theory and has as group of operators a certain finite group Γ. If Γ is solvable, it is shown by an explicit reduction that the group of group extensions is isomorphic to Schur's multiplicator for the group Γ. (Received January 28, 1941.)

118. Marshall Hall: *A problem in partitions.*

Non-void subsets $a_1, \cdots, a_n$ of a set $S(x_1, \cdots, x_n)$ determine a partition matrix

(a_{ij}) in which $a_{ij}=0$ or 1 according as a_i and a_j overlap or not. The question has arisen as to whether every n^2 matrix (a_{ij}) in which $a_{ii}=1$, $a_{ij}=a_{ji}=0$ or 1 is the partition matrix of n subsets of n objects. This is shown to be true for $n=1, 2, 3, 4$, but not for $n\geqq 5$. It is shown that for $n\geqq 5$ every such matrix is a partition matrix but may require as many as $n^2/4$ (n even) or $(n^2-1)/4$ (n odd) objects in S. These values are "best possible" and it is shown that a matrix requiring the maximum number of objects is unique to within an equivalence amounting to renumbering $a_1, \cdots, a_n$. (Received January 24, 1941.)

119. E. R. Kolchin: *On the basis theorem for differential systems.* Preliminary report.

The basis theorem of J. F. Ritt and H. W. Raudenbush for systems of differential polynomials is proved by a new method making no use of ascending sets, but using the axiom of choice. The new proof justifies its existence by the greater generality of its results. For example, in addition to the systems for which the theorem has already been proved, systems of forms with coefficients in a differential field of characteristic zero, the theorem is proved for systems of forms with coefficients in the ring of integers, coefficients in the ring of integral polynomials in x, and coefficients in the field of integers modulo a prime. (Received January 25, 1941.)

120. Howard Levi: *On the ideal theory and structure of differential polynomials.*

In the first part of this paper the differential ideal Σ generated by the form y^p is investigated. It is understood that p is a positive integer greater than unity, and that the underlying ring is that of all differential polynomials in the unknown y whose coefficients are in some differential domain of integrity which contains the rational numbers. Necessary and sufficient conditions for the membership of a form in Σ are obtained. These results are used to show that the criteria (sufficient) of J. F. Ritt for the essentiality of manifolds (Annals of Mathematics, (2), vol. 37 (1936), pp. 556–560, and American Journal of Mathematics, vol. 60 (1938), pp. 5–14) are valid in the abstract case. Extensions are made to cases not discussed by him. Finally the differential ideal generated by the form uv in the ring of forms in the unknowns u and v is examined. Necessary and sufficient conditions for membership in this ideal are obtained. In particular it is shown that for all positive integers r and s the form $u^r v_r^s$ is not in the ideal (v_r being the rth derivative of v). This implies that the ideal in question has no representation as the intersection of differential ideals. (Received January 24, 1941.)

121. Oystein Ore: *Theory of monomial groups.*

If $\{x_i\}$ denotes a set of variables and H some arbitrary group, then the set of all linear transformations $x_i \to h_i x_j$ where the factors h_i belong to H, form a group called the complete monomial group. The present paper contains a study of these groups and gives among other results the determination of all normal subgroups and the group of automorphisms. (Received January 23, 1941.)

122. Oystein Ore: *Transformation of sets.*

The set of all one-to-one correspondences of a set S to itself form a group Σ_s. It is shown that for an arbitrary infinite set this group is complete, that is, all automorphisms are inner automorphisms. (Received January 23, 1941.)

123. P. C. Rosenbloom: *Post algebras*: I. *Postulates and general properties.*

Post algebras are the algebras which bear the same relation to the n-valued logics defined by Post (American Journal of Mathematics, vol. 43 (1921), pp. 180–185) as Boolean algebras bear to the usual 2-valued logic. They are investigated here purely from the algebraic standpoint with no consideration of their interpretation as logic. The first postulate sets for these systems are introduced and the most important general properties are deduced. A fundamental theorem analogous to the Boolean expansion of functions in normal form is proved. The definition of "prime elements" is analogous to Huntington's definition for the Boolean algebras. "Powers of primes" are also defined. A theorem analogous to the fundamental theorem of arithmetic is proved to the effect that "in a Post algebra with a finite number of elements, every element is uniquely factorable into a product of powers of primes, disregarding order and repetition of factors." An arithmetic interpretation generalizing Sheffer's "Boolean numbers" is given. Several unsolved problems are discussed. (Received January 17, 1941.)

124. Ernst Snapper: *Structure of linear sets.*

It is shown that the linear sets of a vector space of arbitrary dimension over an integral domain in which every ideal has a finite basis admit a Noether decomposition into "primary" linear sets. The "associated prime ideals" of the largest primary components are uniquely determined invariants of the linear set. The proofs are based on definitions of "quotient ideal" of a linear set by a linear set, of "quotient linear set" of a linear set by an ideal, and of "product linear set" of an ideal and a linear set. The quotient ideal of a linear set by the whole space is called its "essential scalar ideal" and is fundamental in the definition of primary linear set. The radical of the essential scalar ideal of a primary linear set is prime and is called its associated prime ideal. The "isolated component linear sets" are uniquely determined by their corresponding prime ideals and the theory becomes the ordinary ideal theory in the case of dimension one. Also, this investigation gives rise to the notions of scalar ideal, almost-primarity, radical and essential radical, closed set, and dense set. (Received January 14, 1941.)

Analysis

125. R. P. Agnew: *On methods of summability and mass functions determined by hypergeometric coefficients.*

Let α, β, γ be complex constants and $\gamma \neq 0, -1, -2, \cdots$. Let $\lambda_n(\alpha, \beta, \gamma)$, $n=0, 1, 2, \cdots$, be the coefficients in the power series expansion $\sum \lambda_n z^n$ of the hypergeometric function $F(\alpha, \beta, \gamma; z)$. Let $(H, \alpha, \beta, \gamma)$ be the Hurwitz-Silverman-Hausdorff method of summability generated by the sequence $\lambda_n(\alpha, \beta, \gamma)$. (See Garabedian and Wall, Transactions of this Society, vol. 48 (1940), pp. 195–201.) Let C_r denote the Cesàro method of order r. For certain ranges of the parameters it is shown that $(H, \alpha, \beta, \gamma) = C_{\alpha-1}^{-1} C_{\beta-1}^{-1} C_{\gamma-1}$ and that $(H, \alpha, \beta, \gamma)$ is equivalent to $C_{\gamma-\alpha-\beta+1}$. These results determine conditions under which $(H, \alpha, \beta, \gamma)$ is regular and $\lambda_n(\alpha, \beta, \gamma)$ is the moment sequence of a regular mass function. (Received December 31, 1940.)

126. E. F. Beckenbach: *On almost subharmonic functions.*

It is shown that certain integral inequalities which, in the case of continuous functions, are known to characterize subharmonic functions and functions whose loga-

rithms are subharmonic, no longer characterize these functions when mere summability is assumed. Instead, the inequalities characterize functions which, in the terminology of Szpilrajn, are almost subharmonic or have almost subharmonic logarithms. (Received January 27, 1941.)

127. Stefan Bergman: *On a class of linear operators applicable to functions of a complex variable.*

The totality of functions obtained from the analytic functions of one complex variable regular at the origin by applying the operator $u=\mathrm{P}(f)=\int_{-1}^{+1}\mathrm{E}(z,\bar{z},t)f[z(1-t^2)/2](1-t^2)^{-1/2}dt$ is called the "class of functions" $\mathfrak{C}(\mathrm{E})$. The author studies the properties of functions $u(z,\bar{z})\in\mathfrak{C}(\mathrm{E})$, where $\mathrm{E}=1+z\bar{z}t^2\mathrm{E}_1(z,\bar{z},t)$. Functions u possess the following property: to every point Z there exists a function $H(z,\bar{z};Z)\in\mathfrak{C}(\mathrm{E})$ such that every u satisfies the functional equation $u(z,\bar{z})=(2\pi i)^{-1}\int_{\mathfrak{a}^1}R(Z;u)H(z,\bar{z};Z)dZ$; $R(Z,u)$ being a certain operator, and $\mathfrak{a}^1$ an arbitrary closed differentiable curve lying in the regularity domain of $u(z,\bar{z})$. The theorems of the theory of analytic functions of one complex variable are classified from a certain point of view, and the duality between theorems for functions of one complex variable and those for the class $\mathfrak{C}(\mathrm{E})$ is studied. In this way the author obtains results on the development of u in series $\sum_{\nu=1}^{\infty}a_\nu u_\nu$ in certain domains, $\{u_\nu\}$ being a special set of functions depending on the domain, and further results on the connection between the coefficients A_{mn} of the development $u=\sum A_{mn}z^m\bar{z}^n$ and properties of u, and so on. For example, functions satisfying a linear partial differential equation of elliptic type form a class $\mathfrak{C}(\mathrm{E})$ corresponding to a particular E. (Received January 22, 1941.)

128. Lipman Bers: *Analytic functions of two complex variables in domains bounded by two analytic hypersurfaces.*

Let $\mathfrak{M}^4$ be a domain of the z_1, z_2-space ($z_i=x_i+iy_i$) bounded by two analytic hypersurfaces whose intersection (the *distinguished boundary surface* of $\mathfrak{M}^4$) forms a closed surface $\mathfrak{F}^2$: $z_i=Z_i(\lambda_1,\lambda_2)$, $0\leq\lambda_i\leq 2\pi$. If certain hypotheses (chiefly about the existence of a suitable set of subdomains of the same type) are made, one can generalize theorems proved for simple special cases of such domains (S. Bergman, Compositio Mathematica, vol. 6 (1939), p. 305; S. Bergman and J. Marcinkiewicz, Fundamenta Mathematicae, vol. 33 (1939), p. 75; L. Bers, Comptes Rendus de l'Académie des Sciences, Paris, vol. 208 (1939), p. 1273 and p. 1475). If U is a biharmonic function in $\mathfrak{M}^4$ which possesses continuous boundary values u on $\mathfrak{F}^2$, then $U(z_1,z_2)=\int_0^{2\pi}\int_0^{2\pi}\Omega(z_1,z_2;\lambda_1,\lambda_2\,u(\lambda_1,\lambda_2)d\lambda_1d\lambda_2$ where Ω depends only upon the domain. Biharmonic functions which satisfy certain conditions (for example all non-negative functions) may be represented in the form: $U(z_1,z_2)=\int_0^{2\pi}\int_0^{2\pi}\Omega(z_1,z_2;\lambda_1,\lambda_2)d\omega_{\lambda_1\lambda_2}$; U possesses a finite non-tangential boundary value in every point of $\mathfrak{F}^2$ where the absolute additive set function ω possesses a finite strong derivative. It follows from these theorems that a function $f(z_1,z_2)$ which may be represented in $\mathfrak{M}^4$ as a quotient of two bounded analytic functions possesses almost everywhere on $\mathfrak{F}^2$ finite non-tangential boundary values. If these boundary values are constant on a set of positive two-dimensional measure, f is constant. (Received January 22, 1941.)

129. R. P. Boas: *A note on functions of exponential type.*

It has been shown by D. V. Widder (Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 657–659) that a function $f(x)$ such that $(-1)^nf^{(2n)}(x)\geq 0$ for $n=0, 1, 2, \cdots$ and $0\leq x\leq 1$ must coincide over (0, 1) with an entire function

which is at most of the first order (and of type at most π if it is of order 1). In this note a simple proof of Widder's result is obtained by integrating $\int f(x) \sin \pi x \, dx$ repeatedly by parts; the integrated terms are all non-negative, and consequently an upper bound for $\int |f^{(2n)}(x)| \sin \pi x \, dx$ follows; from this an adequate estimate of $|f^{(2n)}(x)|$ itself is easily obtained. The same method leads to simple proofs of theorems of I. J. Schoenberg (this Bulletin, vol. 42 (1936), pp. 284–288) and J. M. Whittaker and H. Poritsky (Whittaker, Proceedings of the London Mathematical Society, (2), vol. 36 (1933), p. 455; Poritsky, Transactions of this Society, vol. 34 (1932), p. 287). (Received January 21, 1941.)

130. H. L. Garabedian: *Hausdorff methods of summation which include all of the Cesàro methods.*

In this paper are exhibited Hausdorff methods of summation which have the rare property of being more effective than all of the Cesàro methods of real orders. The regular mass functions associated with these Hausdorff methods are notably $\phi_\alpha(u) = 1 - e^{\alpha u/(u-1)}$, $\alpha > 0$, and $\phi_\beta(u) = A \int_0^u (1-t)^{\beta-2} e^{t/(t-1)} dt$, $\beta \geqq 0$, where A is a normalizing factor such that $\phi_\beta(1) = 1$. The conjecture is made that summability $[H, \phi_\beta(u)]$ increases in efficiency with increasing β. (Received December 26, 1940.)

131. Abe Gelbart: *On the behavior of a function of two complex variables in the neighborhood of an isolated essential singularity.*

In this paper the author obtains certain upper and lower bounds for classes of functions of two complex variables in the neighborhood of an isolated essential singularity. Given a sequence of functions $\{h_\nu(z_1, z_2)\}$ regular in a finite four-dimensional domain $\overline{\mathfrak{M}}^4$ such that the analytic surfaces $\{h_\nu(z_1, z_2) = 0\}$ converge to the surface $z_1 = g(z_2)$ which passes through the domain $\mathfrak{M}^4$. It is assumed that the analytic surfaces $h_\nu(z_1, z_2) = 0$ converge to $z_1 = g(z_2)$ with sufficient rapidity. With the aid of theorems of the Runge type for functions of two complex variables for multiply-connected four-dimensional domains (see abstract 46-9-418), the author obtains an upper bound for the moduli of functions of two complex variables belonging to a class of functions analytic in $\overline{\mathfrak{M}}^4 \equiv \mathfrak{M}^4 - G^4$, $G^4 \equiv E[z_1 = g(z_2) + \rho e^{i\lambda}, 0 \leqq \lambda \leqq 2\pi, 0 < \rho < \epsilon]$, and having the same zeros as $\prod_{\nu=1}^{\infty} h_\nu(z_1, z_2)$. Using a technique similar to that of Bergman (Mathematische Annalen, vol. 109 (1934), pp. 324–348), upper and lower bounds are obtained for an arbitrary function meromorphic in the neighborhood of the analytic surface $z_1 = g(z_2)$ and having the same zeros and poles as $\prod_{\nu=1}^{\infty} h_\nu(z_1, z_2)/p_\nu(z_1, z_2)$ wherein $p_\nu(z_1, z_2)$ are functions regular in the domain $\overline{\mathfrak{M}}^4$. (Received January 23, 1941.)

132. H. H. Goldstine: *The prolongment and representation of multilinear functionals.*

It is first shown that every Banach space is linearly equivalent to a linear subset of the space of all bounded and real-valued functions defined on a suitably chosen range. Every linear continuous functional on this space of functions is representable by a Lebesgue-Stieltjes integral. Making use of this theorem, the Hahn-Banach theorem can be proved for multilinear, continuous functionals; that is, the range of definition of every such functional can be extended to the entire space without changing the norm of the functional. At the same time it is shown that these functionals are representable as generalized Fréchet integrals. (Cf. Goldstine, *Bilinear functionals on the space of bounded, measurable functions*, this Bulletin, vol. 43 (1937), pp. 528–531.) (Received December 16, 1940.)

133. P. R. Halmos: *On the decomposition of measures.*

If S is a strictly separable measure space and A is a strictly separable Borel field of measurable sets then, except possibly for a set (in A) of measure zero, S is a direct sum of measure spaces Y_x formed with respect to a measure space X in such a way that the Borel field of all measurable sets depending on x alone coincides with the given Borel field A. This general decomposition theorem yields an easy proof of a result of von Neumann on the decomposition of an arbitrary measure preserving transformation into ergodic parts. Another application of the theorem is to the spectral theory of measure preserving transformations: it is possible by means of it to construct examples of transformations with mixed spectrum—that is, transformations having neither pure point spectrum nor pure continuous spectrum—and to decompose, at least partially, an arbitrary transformation into transformations with pure spectrum. (Received January 22, 1941.)

134. D. H. Hyers: *On the stability of the linear functional equation.*

Let E and E' be linear normed spaces and let δ be a positive number. A transformation $f(x)$ of E into E' will be called δ-linear if $\|f(x+y)-f(x)-f(y)\| < \delta$ for all x and y in E, and linear if $f(x+y)=f(x)+f(y)$. S. Ulam has proposed the following problem of the "stability" of the last equation: Does there exist for each $\epsilon>0$ a $\delta>0$ such that to each δ-linear transformation $f(x)$ there corresponds a linear transformation $g(x)$ satisfying the inequality $\|f(x)-g(x)\| \leqq \epsilon$ for all x in E? In this paper this question is answered in the affirmative for the cases where E and E' are finite dimensional spaces or real Hilbert space, and where $f(x)$ is continuous. It is shown that δ may be taken equal to ϵ in these cases. As Ulam has pointed out, the problem has many generalizations to be investigated. (Received January 6, 1941.)

135. H. N. Laden: *On Kryloff-Stayermann interpolation.*

This paper deals with an interpolation polynomial $F_n(x)$ of degree less than or equal to $4n-1$ which, at the preassigned abscissas $x_1, x_2, \cdots, x_n$ takes preassigned values $y_1, y_2, \cdots, y_n$ respectively and is such that $F_n'(x_i)=F_n''(x_i)=F_n'''(x_i)=0$, $(i=1, 2, \cdots, n)$. If the abscissas are the zeros of the nth Jacobi polynomial and $y_i=f(x_i)$, $(i=1, 2, \cdots, n)$, where $f(x)$ is an arbitrary function continuous on $[-1, +1]$, it is shown that $F_n(x)$ converges to $f(x)$ as $n\to\infty$ uniformly on any closed interval wholly inside $[-1, +1]$. The convergence is uniform on $[-1, +1]$ if proper restrictions are placed on the parameters α, β of the general Jacobi polynomial. Similar results are obtained for interpolation on an infinite interval by means of Laguerre and Hermite polynomials. The first part of the results provides a correction and extension of a theorem of Kryloff and Stayermann. The methods employed are due to Fejér, Shohat, and Szegö. (Received January 11, 1941.)

136. Dorothy Maharam: *On measure in abstract sets.*

Let m be a Boolean σ-ring of sets; n, a σ-ideal in m; Φ, a class of σ-isomorphisms between principal ideals in m/n. It is assumed that Φ contains all products and inverses of its elements. Define the elements of m (n) to be measurable (null) sets, the isomorphisms of Φ to be measure-preserving transformations, and the measure of an element a of m/n to be the totality of division images of a under Φ. The element a is bounded if a is not division-equivalent to any proper sub-element of itself. The set C of bounded measure values is then described; in particular, conditions are given under which C is isomorphic to a set of positive numbers. (Received January 22, 1941.)

137. Szolem Mandelbrojt: *On a Dirichlet series.*

This paper gives necessary and sufficient conditions on $M(\sigma)$ such that there exist a function $f(s)=\sum a_n n^{-s}$, $f(s)\neq 0$, $f(-2q)=0$ $(q\geqq 1)$, $\max|f(\sigma+it)|<M(\sigma)$. (Received December 16, 1940.)

138. N. M. Oboukhoff: *On the validity of total differential as the principal part of the increment of a function of two or more variables.*

K. Weierstrass defined total differential as principal part of the increment of a function. Let $du=Adx+Bdy$ where A and B are finite functions of x and y only, $|dx|$ and $|dy|$ being infinitely small or just small enough relative to $|A|$ and $|B|$. If the terms are all positive or negative, du retains its rank of principal part. However there are cases in which this principal part degenerates. Suppose $du=Adx-Bdy$ and $Adx>Bdy>0$ or $Bdy>Adx>0$. Let $|Adx-Bdy|=K(Adx+Bdy)$, where $0\leqq K\leqq 1$. If K is finite or great enough relative to $|dx|$ and $|dy|$ then du does not degenerate. However it does if K vanishes or is of the same order of magnitude as $|dx|$ and $|dy|$. Thus $K=\big||dx/dy|\,|A|-|B|\big|:(|dx/dy|\,|A|+|B|)$; du degenerates if $|dx/dy|=|B/A|\pm\eta$, where positive η either vanishes or is of the same order of magnitude as $|dx|$ and $|dy|$. For all other values of $|dx/dy|$ from zero to infinity (where dy vanishes) principal part holds. The favorable cases are greatly predominant. Closely similar results obtain for functions of several variables. (Received December 5, 1940.)

139. George Polya: *On converse gap theorems.*

Let $\lambda_1, \lambda_2, \lambda_3, \cdots$ *be integers,* $0<\lambda_1<\lambda_2<\cdots$. *The* necessary *and sufficient condition that* each *analytic function defined by a series of the form* $a_1z^{\lambda_1}+a_2z^{\lambda_2}+\cdots+a_nz^{\lambda_n}+\cdots$ *having a finite radius of convergence be uniform and its domain of definition be simply connected is that* (1) $\liminf_{n\to\infty} n\lambda_n^{-1}=0$. The much deeper half of this theorem concerning the sufficiency of the condition (1) has been proved previously (Annals of Mathematics, (2), vol. 34 (1933), pp. 731–777). The necessity of the condition (1) may be shown by the series (2) $\sum_{m=1}^{\infty}\mathcal{J}(m)z^m/m=\sum_{n=1}^{\infty}(-1)^{\lambda_n}z^{\lambda_n}/\lambda_n^2\mathcal{G}'(\lambda_n)$ where $\mathcal{J}(z)$ and $\mathcal{G}(z)$ are defined by $\pi z\mathcal{J}(z)\mathcal{G}(z)=\sin\pi z$, $\mathcal{G}(z)=\prod_{n=1}^{\infty}(1-z^2/\lambda_n^2)$. If (1) is not fulfilled, the analytic continuation of the series (2) is infinitely many valued (for proving this use Mathematische Zeitschrift, vol. 29 (1929), pp. 604–608). An analogous theorem has been proved by the author in the Comptes Rendus de l'Académie des Sciences, Paris, vol. 208 (1939), pp. 709–711. (Received January 20, 1941.)

140. Tibor Radó: *On the semi-continuity of double integrals in the calculus of variations.*

The purpose of this paper is to extend the results of McShane (*On integrals over surfaces in parametric form*, Annals of Mathematics, (2), vol. 34 (1933), pp. 815–838) to a wide class K of surfaces which may be characterized as follows: A surface S, of the type considered by McShane, belongs to the class K if it admits of a representation where its Lebesgue area is given by the usual integral formula. All the results of McShane are shown to remain valid for this class K. (Received December 16, 1940.)

141. Raphaël Salem: *On absolute convergence of trigonometrical series.* I.

The following theorems are proved: If the series (S) $\sum\rho_n\cos(nx-\alpha_n)$, $(\rho_n\geqq 0)$ converges absolutely at two points x_0, x_1, the series $\sum\rho_n|\sin n(x_1-x_0)|$ converges. If

$\sum \rho_n = \infty$ the series (S) cannot converge absolutely at more than one point if $\rho_{n+1} \leqq \rho_n$ or even if ρ_{n+p}/ρ_n is bounded ($p>0$). If $\sum \rho_n = \infty$, the series $\sum \rho_n \cos(k_n x - \alpha_n)$ cannot converge absolutely at two points whose abscissae differ from δ if the numbers $k_n\delta$ are uniformly distributed (mod 2π). If $\sum \rho_n = \infty$ and the series (S) converges absolutely in a perfect set N, then for every bounded function F non-decreasing, constant in every interval contiguous to N but increasing from one interval to another, the cosine Fourier-Stieltjes coefficients of even rank of dF have their upper limit equal to $F(2\pi) - F(0)$. This condition is sufficient to ensure the absolute convergence of (S) "almost everywhere" in N, with $\sum \rho_n = \infty$. Further properties of sets of the type N are discussed. (Received January 30, 1941.)

142. Raphaël Salem: *On absolute convergence of trigonometrical series.* II.

The following extension of the Denjoy-Lusin theorem is proved: The set for which the ratio (I) $\sum_1^n \rho_n |\cos(nx-\alpha_n)| / \sum_1^n \rho_n$ has its upper limit less than or equal to α is of measure zero if $\alpha < 2/\pi$. Moreover any perfect set P in which the ratio (I) has its upper limit less than or equal to α less than $2/\pi$ has the property that every bounded function F, non-decreasing, constant in every interval contiguous to P, but not everywhere, is such that the Fourier-Stieltjes coefficients of dF do not tend to zero. The constant $2/\pi$ is the best possible one; in fact, if $\rho_n = O(1)$, the ratio (I) tends to $2/\pi$ almost everywhere. (Received January 30, 1941.)

143. I. J. Schoenberg: *On integral representations of completely monotone and related functions.*

In a recent paper W. Feller proved very elegantly the chief results of S. Bernstein and Widder concerning the nature of functions $f(x)$ allowing a Laplace-Stieltjes representation (1) $f(x) = \int_0^\infty e^{-xt} d\alpha(t)$ with various restrictions as to $\alpha(t)$. In the first part of the paper, Widder's original tools are used for the purpose of an equally short presentation of this theory. In particular, Widder's inversion formula for (1) appears in a very natural way. In the second part a similar theory is developed for the representation (2) $f(x) = \int_0^\infty K_n(xt) d\alpha(t)$ ($x>0$; n a positive integer) where $K_n(x) = (1-x)^n$ if $0 \leqq x \leqq 1$, $K_n(x) = 0$ if $x > 1$. Since $K_n(x/n) \to e^{-x}$ as $n \to \infty$, (1) is seen to be a limiting case of (2). The results may be described very roughly as follows: The role played by the *completely monotone functions* $f(x)$ (that is, satisfying the conditions (3) $(-1)^n f^{(n)}(x) \geqq 0$ ($x>0$; $n = 0, 1, 2, \cdots$)) in the theory of the representation (1), is taken over in the case of (2) by the *$(n+1)$-times monotone functions* $f(x)$. Here a function is called $(n+1)$-times monotone if $f(x) \geqq 0$, $-f'(x) \geqq 0$, $f''(x) \geqq 0, \cdots$, $(-1)^{n-1} f^{(n-1)}(x) \geqq 0$, ($x>0$), the last function $(-1)^{n-1} f^{(n-1)}(x)$ being also required to be non-increasing and convex for $x>0$. (Received January 27, 1941).

144. W. T. Scott and H. S. Wall: *The transformation of series and sequences.*

The transformation $S_n = \sum_{p=0}^n (1/2)^{2n+1} C_{2n+2,n-p} \cdot s_p$, ($n = 0, 1, 2, \cdots$), which arises from the power series transformation $\frac{1}{2}(1-x)f(x) = F(z)$, $z = 4x/(1-x)^2$, is shown to define a regular method of summation, that is, $s_p \to s$ implies $S_n \to s$. Let this be called Schur summability and denoted by the letter S. It is proved that $S \supset VP$, but $VP \not\supset S$, where VP denotes de la Vallée Poussin summability; S sums the geometrical series in precisely the region in which VP sums this series; $S \approx M$, where M is the summability

introduced by Mersman (this Bulletin, vol. 44 (1938), pp. 667–673). It is shown that if $\phi(u)\in BV[0, 1]$, and $g(x)=\int_0^1 d\phi(u)/(1+xu)$, then the sequence $\{g(n)\}$, $(n=0, 1, 2, \cdots)$, is a *C-Folge* in the sense of Hausdorff, and the Hausdorff mean $[H, g(n)]$ is regular if and only if $\phi(1)-\phi(0)=1$. If $\phi(u)$ is real and monotone and $\phi(1)-\phi(0)=1$, then $H, g(n)\subset(C, 1)$, and is equivalent to convergence if and only if the function $g(x)$ is bounded away from zero in the half-plane $R(x)>-\frac{1}{2}$. Let $\theta(u)=\sum_{p=1}^{\infty}q_p u^p$, $q_p\geqq 0, \sum q_p<\infty$. Then $a_n=1+n\int_0^1 u^n d\theta(u)$, $(n=0, 1, 2, \cdots)$, defines a Hausdorff mean $[H, a_n]$ equivalent to convergence if and only if $\sum nq_n<\infty$. (Received January 24, 1941.)

145. J. A. Shohat: *On Bernoulli numbers and polynomials.*

Certain relations are established between Bernoulli polynomials $B_n(x)$ and those of Legendre and symmetric polynomials of Jacobi; and a new proof is given of Jacobi's theorem concerning the expression of $B_{2n}(x)$ in powers of $x(1-x)$. A reasoning of Mandl is utilized and made rigorous in order to obtain in a new way the well known remarkable relation between the Bernoulli number B_{2n} and the zeta-function $\zeta(2n)$. Applications are made to $S_n(p)=1^p+2^p+\cdots+(n-1)^p$. (Received January 8, 1941.)

146. S. E. Warschawski: *On conformal mapping of infinite strips.*

Let S denote the strip $\phi_-(u)<v<\phi_+(u)$, $-\infty<u<+\infty$ ($\phi_+(u)$, $\phi_-(u)$ continuous), in the w-plane, $w=u+iv$. Let $z=Z(w)$ ($\lim_{u\to+\infty}\Re Z(w)=+\infty$) map S conformally onto the strip $|y|<\pi/2$ of the z-plane, $z=x+iy$. The principal object of this paper is to obtain asymptotic expressions for $Z(w)$ and its derivative $Z'(w)$ as $u\to+\infty$. For this purpose two inequalities are established, which are similar to those of L. Ahlfors (Acta Societatis Scientiarum Fennicae, new series A, vol. 1 (1930) p. 10 and p. 16), but which, due to some assumptions regarding the smoothness of the boundary of S, yield sharper estimates for large values of $\Re w_1$ and $\Re w_2$. The asymptotic expressions for $Z(w)$ and $Z'(w)$ are then applied, after suitable transformations, to the study of the order of magnitude of the mapping function of a region R onto a circle in a neighborhood of a finite boundary point P of R. These applications include the cases where P is the vertex of a corner, or of a cusp, or is the asymptotic point of two "concurrent" spirals, and contain as special cases the results presented by the writer in two previous abstracts (42-5-219 and 42-11-409). (Received December 9, 1940.)

Applied Mathematics

147. E. S. Allen and Harvey Diehl: *The enumeration of glycols.*

Recently the authors extended the method of Henze and Blair for enumerating certain organic compounds; in particular, the alcohols. This work is used as a basis for a recursive method of enumerating isomeric glycols, that is, compounds whose molecules have two oxygen atoms. The basic numbers which result are the number of structurally asymmetric glycols possessing n carbon atoms and having α enantiomorphically distinct forms, and the number of structurally symmetric glycols possessing n carbon atoms and having α distinct forms, of which β are completely symmetric. In all cases symmetry indicates identity of aspect of the molecule when viewed from the two hydroxyl radicals. (Received December 31, 1940.)

148. Stefan Bergman: *Numerical methods for conformal mapping of polygonal domains.*

The problem of the approximate determination of a conformal mapping can be reduced in certain cases to numerical operations which can be carried out with the aid of devices for computation already in existence or by electrical calculating machines combined, perhaps, with punch machines. In the present paper the calculation of the constants (that is, the branch points of the integrand) in the Schwarz-Christoffel formula (which transforms the half-plane into a polygonal domain) is reduced to the determination of the Fourier coefficients of $[R(\phi)]^n$, and to the solution of a system of linear equations. If the domain is a star domain, $R=R(\phi)$ is the equation of its boundary curve in polar coordinates. Further, methods for the calculation of the resulting integrals are discussed. The method is applied to a technical problem, namely to the problem of torsion in a beam with a polygonal section. The analogous method can be applied for the solution of boundary problems and has important applications in certain problems of aeronautics. (Received January 22, 1941.)

149. F. H. Clauser: *Exact solutions of the equations for the flow of a compressible fluid.* Preliminary report.

Several solutions of the equations given by Tschaplygin for the flow of a compressible fluid are discussed and a method presented for easily accomplishing the transformation of the solutions in the hodograph plane back to the physical plane. (Received December 18, 1940.)

GEOMETRY

150. L. M. Blumenthal and G. E. Wahlin: *On the spherical surface of smallest radius enclosing a bounded subset of n-dimensional euclidean space.*

A short elementary proof is given for the theorem: *If M is any bounded subset of n-dimensional euclidean space E_n with positive diameter d, then there is a unique $(n-1)$-dimensional spherical surface of smallest radius r enclosing M, and $r \leqq [n/2(n+1)]^{1/2} \cdot d$.* In a proof abounding with algebraic difficulties, H. W. E. Jung established these results in 1901 for the case of finite point sets and indicated their extension to infinite sets at the end of his long paper (Journal für die reine und angewandte Mathematik, vol. 123 (1901), pp. 241–257). The simplification offered by the present proof is afforded in large measure by a lemma which shows that *if each $n+1$ points of a subset M in E_n may be enclosed by an $(n-1)$-dimensional spherical surface of radius r then M itself has this property.* The proof exhibits the geometrically simple nature of the theorem. (Received January 24, 1941.)

151. J. J. DeCicco: *Equilong geometry of differential equations of first order.*

With this paper the study of the equilong geometry of a field of lineal elements is begun. This may be considered to be an analogue of a preceding paper by Kasner and the author in which the conformal geometry of a field is developed. As defined by Kasner, a dual-isothermal family consists of ∞^1 curves which are equilongly equivalent to a pencil of circles (all those tangent to two fixed lines, distinct or coincident). Obviously all dual-isothermal families are equilongly equivalent. It is found that any

non-dual-isothermal family possesses two absolute differential covariants of the third order. Conversely if two fields possess the same two covariants, then there exist $2\infty^2$ equilong correspondences which will carry one field into the other. For any two nearby nonparallel elements of the field $w=w(u, v)$, the general equilong distance is defined by $ds^2=w_v{}^2du^2$, whereas for two nearby parallel elements, this distance is $ds^2=w_v{}^2dv^2$. The final result is that in equilong geometry any absolute differential covariant of a field is a function of two fundamental covariants and their derivatives with respect to the equilong arc lengths of the unions and the equiparallel series of the field. (Received January 20, 1941.)

152. J. J. DeCicco: *Isodeviate systems of geodesic series.*

This paper is concerned with the derivation of some further results in the differential geometry of a field of lineal elements in the plane. A system of ∞^1 series of a given field, which corresponds by an equideviate transformation to a linear pencil of turbines of a flat field, is called an isodeviate system. If a field possesses at least one isodeviate system of geodesic series, then it must possess ∞^2 such systems. The condition necessary and sufficient for this is that the gaussian curvature K be the same at all parallel elements of a field. To compare this with the corresponding theory in the geometry of surfaces in euclidean three-space, see Kasner, *Isothermal systems of geodesics*, Transactions of this Society, vol. 5 (1904), pp. 56–60. (Received December 31, 1940.)

153. J. J. DeCicco: *Lineal element transformations which preserve dual-isothermal families.*

This paper is analogous to the one by Kasner and the author in which the group of all lineal element transformations which preserve isothermal families is determined. Kasner has defined a dual-isothermal family to consist of ∞^1 curves which are equilongly equivalent to a pencil of circles (in line geometry). In this work all lineal element transformations are found which preserve dual-isothermal families. In hessian or equilong coordinates this group is $U=\phi$, $V=(a_1v+b_1w+c_1)/(av+bw+c)$, $W=(a_2v+b_2w+c_2)/(av+bw+c)$, where ϕ, a, b, c, a_1, b_1, c_1, a_2, b_2, c_2 are functions of u only. The subgroup of contact transformations is $U=\phi(u)$, $V=v\psi(u)+\chi(u)$, obviously larger than the equilong group. Earlier the author had found that a non-dual-isothermal family possesses two absolute differential covariants of the third order. In the final part of the present paper, it is proved that any lineal element transformation which preserves both of these two covariants must be an equilong transformation. (Received January 21, 1941.)

154. J. J. DeCicco: *The differential geometry of the Laguerre group G_6.*

In the Laguerre inversive geometry of the plane the length of arc of a curve is defined in equilong coordinates (x, y) by $s=\int(y''')^{1/2}dx$. The Laguerre inversive curvature is $K=[4y'''y^v-5(y^{iv})^2]/4(y''')^2$. The fundamental result of the paper is that two curves are equivalent under the Laguerre group G_6 if and only if their curvatures are the same functions of the arc length. Thus $K=F(s)$ is the intrinsic equation of any curve. As the Laguerre group of the complex plane is isomorphic to the group of rigid motions of complex euclidean three-space with the curves of the plane corresponding to the minimal curves of spaces, the fundamental result also furnishes the intrinsic equations of the minimal curves of space. (Received December 31, 1940.)

155. J. M. Feld: *The geometry of whirls and whirl-motions in space.*

The geometry of the six-parameter whirl-motion group G_6 in the plane had its origin in Kasner's fundamental paper, *The geometry of turns and slides, and the geometry of turbines* (American Journal of Mathematics, vol. 33 (1911), p. 193). In a recent paper by Feld, *Whirl-similitudes, euclidean kinematics, and non-euclidean geometry* (this Bulletin, abstract 46-5-270), Kasner's G_6 was extended first to a mixed six-parameter group composed of eight distinct continuous families, one of which is G_6, and second to a seven-parameter mixed group of eight families—the *group of whirl-similitudes*. The latter group was shown to be isomorphic with the group of automorphisms of *pseudo-elliptic three-space*. Feld also presented a (1, 1) mapping of lineal elements, flat fields, and turbines on planar euclidean displacements, symmetries, and ordered pairs of points respectively. The subject of this paper is the development of an analogous theory for three-space, where a mixed eight-family twelve-parameter continuous group of whirl-motions plays the principal role. Quaternion geometry is employed. (Received January 14, 1941.)

156. Aaron Fialkow: *The foundations of the conformal differential geometry of a subspace.*

For an arbitrary subspace V_n in any Riemann space V_m $(0<n<m;\ m>2)$, the author shows that it is possible to define a system of differential forms, termed the *conformal fundamental forms* (c.f.f.'s) of V_n, which enjoy the following properties: (1) If $V_n \subset V_m$, $\overline{V}_n \subset \overline{V}_m$ and $V_m \leftrightarrow \overline{V}_m$, $V_n \leftrightarrow \overline{V}_n$ by a conformal map, then the c.f.f.'s of V_n and $\overline{V}_n$ are equal. (2) Conversely, if V_m and $\overline{V}_m$ are conformally euclidean spaces and the c.f.f.'s of V_n and $\overline{V}_n$ are equal, then a conformal transformation exists so that $V_m \leftrightarrow \overline{V}_m$, $V_n \leftrightarrow \overline{V}_n$. (3) A V_n exists with any preassigned c.f.f. whose coefficients satisfy certain conformally invariant partial differential equations analogous to the classical generalized Gauss-Codazzi equations. The results for $n \geqq 4$ are typical while the cases $n=3$, 2, and 1 respectively exhibit increasing degrees of deviation from the normal situation. These theorems serve as the basis for the development of a conformal differtial geometry in many ways similar to classical differential geometry but also exhibiting a number of essential differences. The principal tool is a new simple type of differentiation (with respect to the subspace) which enjoys all the usual properties of covariant differentiation as well as a number of others which give it its distinctive conformal character. (Received January 25, 1941.)

157. M. C. Foster: *Note on autopolar surfaces.*

This paper is concerned with surfaces autopolar with respect to the paraboloid $2z=x^2+y^2$. Such autopolar surfaces are considered as special solutions of those partial differential equations which are invariant under the dual transformation for which the above paraboloid is the quadric of reference. Various metric properties are considered. (Received January 22, 1941.)

158. Edward Kasner and J. J. DeCicco: *Conformal geometry of differential equations of first order.*

In this paper the conformal geometry of nonisothermal fields of lineal elements in the plane is studied. A field F possesses two absolute conformal covariants of the third order. Conversely, if two fields F and G possess these same covariants at corresponding elements, then there exist $2\infty^2$ conformal transformations which will carry one into the other. These results may be interpreted geometrically as follows. If the

equation of the field F is $\theta=\theta(x, y)$, where θ is the inclination, the conformal distance dS between two lineal elements of F is defined by $dS^2=\frac{1}{4}(\theta_{xx}+\theta_{yy})(dx^2+dy^2)$. With this definition of distance and the usual notion of angle, it results that the two covariants are the geodesic curvatures of the unions and the orthogonal series of F. All other covariants of F are functions of these two geodesic curvatures and their partial derivatives with respect to the conformal arc lengths of the unions and the orthogonal series. In the final part of the paper, many interesting conformal properties of n-webs of curves are obtained. (Received December 31, 1940.)

159. Edward Kasner and J. J. DeCicco: *Conformal geometry of velocity systems.*

In this paper the conformal geometry of velocity systems is studied. A set of ∞^2 curves is a velocity system if and only if the ∞^1 osculating circles of the ∞^1 curves of the set passing through a fixed point, constructed at this point, form a pencil. Any arbitrary point transformation for which neither of the two families of minimal lines is preserved carries exactly one velocity system into a velocity system. Any point transformation which preserves only one family of minimal lines converts ∞^1 velocity systems into velocity systems. Finally, the conformal group preserves all velocity systems. A velocity system may contain exactly ∞^2, ∞^1, one, or no isothermal systems. A velocity system is called a Γ family if it is conformally equivalent to the ∞^2 circles orthogonal to a fixed circle. A velocity system is a Γ family if and only if it possesses ∞^2 isothermal families. All the conformal covariants of a velocity system are obtained. Finally, the reciprocal system of any Γ system is treated. (Received December 31, 1940.)

160. Edward Kasner and J. J. DeCicco: *Infinite groups generated by equilong involutions and symmetries.*

In equilong geometry, the set of all equilong transformations of period two may be classified into three distinct types: equilong involutions, K symmetries, and D inversions. (Proceedings of the National Academy of Sciences, vol. 26 (1940), pp. 471–476.) In the present paper the infinite groups generated by these transformations are determined. The group K'_{sym} formed by all K symmetries consists of K symmetries and K translations (products of two K symmetries). Any transformation of the group G'_{involi} (D'_{invers}) formed by all equilong involutions (D inversions) can be factored into involutions (D inversions) in an infinitude of ways of which at least one can be factored into four or fewer involutions (D inversions). The fundamental result is that the group generated by all equilong transformations of period two is identical with the group generated by K symmetries and D inversions. To contrast this with the conformal theory, see Kasner, American Journal of Mathematics, vol. 38 (1916), pp. 177–184. (Received December 31, 1940.)

161. Edward Kasner and J. J. DeCicco: *Lineal element transformations which preserve isothermal families.*

In this paper the authors seek to generalize the well known result that the conformal transformations are the only *point correspondences* which carry every isothermal family into an isothermal family of curves. They find the group of all *lineal element transformations* of the plane (not necessarily of the contact type) which preserves all isothermal families. This group is given in (x, y, θ), where (x, y) are the car-

tesian coordinates of the point, and θ is the inclination of the element, by $X=\phi(x, y)$, $Y=\psi(x, y)$, $\Theta=k\theta+h(x, y)$, where k is a nonzero constant, ϕ and ψ satisfy the Cauchy-Riemann equations (direct or reverse), and h is any harmonic function. The only contact correspondences are the conformal ones. In the authors' previous work, it was shown that any non-isothermal field possesses two absolute differential covariants of the third order under the conformal group. In the final part of the present paper it is proved that a lineal element transformation which preserves either one of these two covariants must be a conformal transformation. (Received January 20, 1941.)

162. Edward Kasner and J. J. DeCicco: *The classification of analytic arcs or elements under the group of arbitrary point transformations.*

This paper begins the study of the invariant theory of a single irregular analytic arc or element based on the infinite group of arbitrary point transformations in the plane. The correspondences are (regular) analytic in x and y. Then y may be written as a power series which proceeds according to positive integral powers of the pth root of x. Let q be the first power of the pth root of x which is not a multiple of p. The index p and the rank q are arithmetic invariants; and all elements with the same p and q form the single species (p, q). Absolute differential invariants exist for all irregular species except in the cases (4, 5), (4, 6), (4, 7), (3, q), and (2, q). The species (4, 5) and (4, 7) may be separated into two and three distinct sets respectively. The species (4, 6) and (3, q) possess an additional arithmetic invariant. All the elements of the species (2, q) are equivalent. The paper will appear in the Proceedings of the National Academy of Sciences. (Received December 12, 1940.)

163. Don Mittleman: *Theory of ortho-family: A generalization of natural family.*

A surface will be said to be mapped orthogonally onto a plane if the image of an orthogonal net on the surface is the rectangular cartesian net of the plane. An ortho-family is defined as the image of the geodesics of the surface under the particular orthogonal mapping. The necessary and sufficient conditions that a two-parameter family of curves in the plane be an ortho-family are given analytically. It is a simple consequence of these conditions that an ortho-family which is a velocity family must be a natural family. Finally, if an ortho-family which is obtained by an orthogonal, non-conformal mapping is a natural family, then the surface whose geodesics are the pre-image of the given natural family is a surface of Liouville referred to the coordinate system for which $ds^2=(1/f-1/g)(dx^2/f+dy^2/g)$, where f is a function of x alone, and g a function of y alone. (Received January 24, 1941.)

164. Nelson Robinson: *On the contact of a quartic surface with a general analytic surface.*

Using the differential equations of a general analytic surface S referred to its asymptotic net, the power series development of S is computed to terms of the sixth degree. By means of this expansion conditions are determined in order that a general quartic surface have various orders of contact with the original surface S at a point P. Special cases of the quartics Q are discussed with particular attention given to the cases where Q is composite, the components being quadric surfaces. Necessary and sufficient conditions are found in order that Q be composed of quadrics of Darboux, or quadrics of Lie. (Received December 30, 1940.)

165. Peter Scherk: *On real closed curves of order $n+1$ in projective n-space.* II. Preliminary report.

In the first part of this paper (abstract 46-11-502) the author discussed differentiable closed curves K^{n+1} of real order $n+1$ in R_n by means of a certain single-valued correspondence of the K^{n+1}. He proved that $S \leqq n+1$, $S \equiv n+1 \pmod 2$ if S is the sum of the multiplicities of the singular points, and he characterized the case $S=n+1$. Extending a simple remark on rotation numbers to multi-valued correspondences, the author discusses a two-valued and a three-valued correspondence defined on certain arcs of the K^{n+1} and on the whole K^{n+1} respectively, and connected with the projections of the K^{n+1} from its osculating $(n-2)$-spaces and $(n-3)$-spaces respectively. The study of these two correspondences yields: (1) the first estimates of the number of osculating $(n-2)$-spaces which meet the K^{n+1} again; (2) the classification of the K^{n+1} with $S=n-1$; (3) the classification of the K^3; (4) a more systematic access to the classification of the K^4 (previously obtained by the author). (Received January 24, 1941.)

166. Alexander Wundheiler: *Abstract algebraic definition of an affine vector space.* Preliminary report.

A linear set over the field of real numbers will be called a simple vector space, and its elements, simple vectors. Two simple vector spaces A and B are cogrediently coupled if for any a in A and b in B a real number $f(a, b)$ is defined, such that $f(ka, b)=f(a, kb)=kf(a, b)$; $f(a, b'+b'')=f(a, b')+f(a, b'')$; $f(a'+a'', b)=f(a', b)+f(a'', b)$. The a's and b's are then contragredient vectors. If A and B are of the same dimension, the set $A+B$ is called an affine vector space, a is a contravariant affine vector, b a covariant one, or vice versa. Various illustrations are given, as electrical networks, the space of fruit juice cocktail cans, and so on. (Received January 24, 1941.)

167. Oscar Zariski: *Pencils on an algebraic variety and a new proof of a theorem of Bertini.*

The theorem of Bertini-Enriques states that if a linear system of W_{r-1}'s on a V_r is reducible (that is, every W_{r-1} of the system is reducible) and is free from fixed components, then the system is composite with a pencil. In this paper a new proof of this theorem is given, together with an extension to irrational pencils. With every pencil $\{W\}$ there is associated a field P of algebraic functions of one variable, a subfield of the field Σ of rational functions on V_r. The essential point of the proof is the remark that $\{W\}$ is composite if and only if P is not maximally algebraic in Σ. The rest of the proof, in the case of pencils, follows from the fact that an irreducible algebraic variety V_r over a ground field K is absolutely irreducible if K is maximally algebraic in Σ. In the case of linear systems of dimension >1, the proof is based on the following lemma: if K is maximally algebraic in Σ and if x_1, x_2 are algebraically independent elements of Σ, then for all but a finite number of elements c in K the field $K(x_1+cx_2)$ is maximally algebraic in Σ. (Received December 12, 1940.)

Logic and Foundations

168. Alvin Sugar: *Postulates for the calculus of binary relations in terms of a single operation.*

In a recent paper (*Postulates for the calculus of binary relations*, Journal of Symbolic Logic, vol. 5 (1940), pp. 85–97) J. C. C. McKinsey gave a set of postulates for the

calculus of binary relations in terms of the two operations $|$ and $<$. In his paper McKinsey shows that $<$ is definable in terms of $|$ but not conversely. In this paper the author develops a set of independent postulates for the calculus of binary relations in terms of the single operation $|$. (Received January 25, 1941.)

STATISTICS AND PROBABILITY

169. Archie Blake: *The exploratory determination of statistical distributions.*

The most powerful statistics are not always the most "efficient" or those whose distributions have already been tabled, but their distributions can be computed in small samples without inordinate labor. To find the distribution of $g(X=x_1, \cdots, x_n)$ subject to the condition $P(X)$, compute requisite values of g^{-1} (multiple-valued) and require $\int_{g^{-1}} f(X)dX$, f being the given distribution function. The chief task is the computation of many values of the functions involved; this is alleviated by modern machine methods (especially punched cards). Tables of f, g, and so on, with their derivatives or the required fractions of the latter are prepared once for all on cards; thereafter the work consists only of interpolation. (Taylor's series recommends itself in this problem, as it converges more rapidly than ordinary interpolation formulae, and in the case of multivariate interpolation is much less complicated.) For a statistic whose asymptotic distribution is known, we can interpolate approximately between this and the results of the computation for small n to obtain an estimated distribution for any n. (Received January 23, 1941.)

170. W. G. Madow: *The distribution of the general quadratic form in normally distributed random variables.*

The distribution of the general quadratic form in normally distributed random variables is obtained. This distribution is used to obtain the distribution of Neyman's estimate in the theory of the representative method of sampling, and it is also used to obtain a generalization of P. L. Hsu's distribution of Student's ratio when the true means and variances are unequal. The distribution is also used in tests occurring in the analysis of variance with non-orthogonal data, and the study of differences of various orders. In the latter use, a test for periodicity is obtained. (Received January 25, 1941.)

171. Henry Scheffé: *An inverse problem in correlation theory.*

An $m \times n$ matrix Y may be used to represent m sets of measurements on n variables. The $n \times n$ matrix R of correlation coefficients r_{ij} is a function of the matrix Y, $R=F(Y)$. Necessary conditions (C) that $R=F(Y)$ are that R be real symmetric with diagonal elements unity, and positive (rank=index). Given any matrix R satisfying the conditions (C), does a "statistics problem Y" exist such that $R=F(Y)$? It is proved by matrix methods that there are no solutions Y with $m \leqq$ rank R, but ∞ solutions for each $m >$ rank R. Particular solutions are constructed and the most general solution is characterized. Some corollaries are drawn. (Received January 8, 1941.)

172. Jacob Wolfowitz: *Tests of statistical hypotheses where the distribution forms are unknown.*

The likelihood ratio criterion for testing composite statistical hypotheses, discovered by Neyman and Pearson and recently proved by Wald to be asymptotically

most powerful, is extended to testing composite hypotheses where the forms of the distribution functions are entirely unknown (continuity is assumed) and where tests must be based on the order relations among the observations. Thus a general method for treating problems of this character is obtained. For the problem of two samples (Wald and Wolfowitz, Annals of Mathematical Statistics, June, 1940) the resultant statistic is $\prod_j l_j^{l_j}/(l_j)!$, where l_j is the length of the jth run. Its logarithm is asymptotically normally distributed. The result is immediately extensible to the problem of k samples. For the problem of independence (Hotelling and Pabst, Annals of Mathematical Statistics, March, 1936) a similar statistic is obtainable which differs from the commonly used rank correlation coefficient. The method used to prove the logarithms of these statistics asymptotically normally distributed is applicable to proving the asymptotic normality of a large class of functions of partitions of an integer, of functions of sequences where the subsequences of odd and even numbered elements are themselves partitions of different integers of fixed ratio, and to similar problems (Received January 13, 1941.)

Theory of Numbers

173. Paul Erdös and Joseph Lehner: *On the distribution of the number of summands in the partitions of a positive integer.*

Let $p_k(n)$ denote the number of partitions of n into not more than k summands. Then for $k=n^{1/2}(\log n/C)+xn^{1/2}$, $C=\pi(2/3)^{1/2}$, $p_k(n)/p(n)\sim\exp\{-2\exp(-Cx/2)/C\}$, where $p(n)$ is the number of unrestricted partitions of n. For $k=o(n^{1/3})$, $p_k(n)\sim C_{n-1,k-1}/k!$, uniformly in k. Let $P(n)$ be the number of partitions of n into different summands. Then for "almost all" partitions, the number of summands in a given partition not exceeding $xn^{1/2}$ lies between $2n^{1/2}\{\log 2/(1+\exp(-Dx))\}/D\pm\epsilon n^{1/2}$, $D=\pi(1/3)^{1/2}$. The methods used are elementary in character. (Received January 23, 1941.)

174. Gordon Pall: *The construction of positive ternary quadratic forms.*

A method is developed of writing down quickly the reduced integral positive ternary quadratic forms of a given determinant, order, or genus. (Received January 30, 1941.)

175. H. A. Rademacher: *Ramanujan's identities under modular transformations.*

If the Ramanujan identities which exhibit the divisibility of $p(5n+4)$ and $p(7n+5)$ by 5 and 7, respectively, are expressed in terms of the Dedekind function $\eta(\tau)$, they can be subjected to modular transformations. Each of the two identities goes over into a new one, which is noteworthy because of the occurrence of the Legendre residue symbol. These new identities lead to a direct proof of the Ramanujan identities. The same procedure can be applied to an identity given by Zuckerman, involving $p(13n+6)$. Other identities, proved by Watson and Zuckerman, lead to modular equations of "level" (Stufe) 5 and 7. (Received January 24, 1941.)

Topology

176. F. B. Jones: *Monotonic collections of peripherally-separable connected domains.*

It is shown that: *In a locally connected metric space, every monotonic collection of*

peripherally-separable connected domains contains a countable subcollection running through it. Weakening the hypothesis of the theorem in any one respect results in a false proposition. A few simple applications are indicated. (Received January 24, 1941.)

177. Shizuo Kakutani: *A generalization of Brouwer's fixed-point theorem.*

A generalization of Brouwer's fixed-point theorem (concerning a continuous point-to-point mapping $\phi(x)$ of a finite dimensional simplex S into itself) is obtained for the case of a point-to-set mapping $\Phi(x)$, when the image $\Phi(x)$ of each point is a closed, convex set contained in or equal to S. Instead of the continuity of $\phi(x)$, the upper semi-continuity of $\Phi(x)$ is assumed. The fixed-point theorems of J. von Neumann in his theory of games (Mathematische Annalen, vol. 100 (1928)) and mathematical economics (Ergebnisse eines Mathematischen Kolloquiums, vol. 8 (1937)) are easy consequences of this result. In order to prove the theorem, the fixed-point theorem of Brouwer must be assumed. (Received January 22, 1941.)

178. J. L. Kelley: *A theorem on transformations.*

Let $f(X)=Y$ be a monotone interior transformation of a compact metric space and suppose dim $Y<+\infty$. Further suppose diam $f^{-1}(y)\geq d>0$ for all $y\in Y$. Then there exists a closed, totally disconnected subset X_0 of X such that $f(X_0)=Y$. As a result it is shown that if there exists a Knaster continuum (that is, an indecomposable continuum every subcontinuum of which is decomposable) of dimension greater than 1, then there exists a Knaster continuum of infinite dimension. (Received December 30, 1940.)

179. R. C. Stephens: *On abstract mappings.*

Following the notation of Chittenden, a space (P, ϕ) denotes an abstract set P and a relation between subsets of P. A mapping T of (P, ϕ) onto (Q, ϕ) will be called a ϕ-mapping provided that for every subset E of Q, the inverse transformation T^{-1} satisfies the relation $\phi T^{-1}(E)=T^{-1}\phi(E)$. In this paper, theorems are established for ϕ-mappings with ϕ having various simple properties. These results are then applied to standard operators. The relationship of ϕ-mappings to interior transformations is shown. (Received January 20, 1941.)

180. Hassler Whitney: *Geometric methods in combinatorial topology.* Preliminary report.

Singular cycles in a polyhedron P give a geometric content to the theory of cycles in a corresponding complex K. "Hypercycles" in P are defined corresponding to cocycles in K. A hyperchain ρ is, essentially, a homomorphism, into a coefficient group G, of singular cycles C whose boundaries do not touch the "nucleus" N_ρ of ρ, and which themselves do not touch the "nuclear boundary" N'_ρ ($N'_\rho \subset N_\rho$). The "hyperboundary" ρ' of ρ is easily defined: $N_{\rho'}=N'_\rho$, $N'_{\rho'}=0$. Any $\rho^r \cdot C^r$ is an element of G. If K' is the first derived of K, to each oriented σ corresponds a ρ_σ, whose nucleus is the set of closed cells of K' touching σ only at its center, and with "orientation" that of σ. Then $\rho_\sigma \cdot \sigma=1$. In an orientable manifold $M^n=K$, a cycle C^r of K corresponds to a ρ^{n-r} with nucleus C^r, and a chosen orientation of cells σ^{n-r} normal to C^r. If ρ^r and ρ^s are in "general position," their "product" gives a $\rho^{r+s}=\rho^r \smile \rho^s$; $N_{\rho^{r+s}}=N_{\rho^r} \cap N_{\rho^s}$. Also $\rho^s \frown C^{r+s}=C^r$. In M^n, passing to cycles, both products give ordinary intersections. (Received January 25, 1941.)

181. G. T. Whyburn: *On mappings into the circle.*

If $f(x)$ is a continuous mapping of a metric space X into the unit circle S of the complex plane, the property of a subset Y of X to have f not equivalent to 1 (in the Eilenberg sense) on Y is inducible. Thus if X is compact and f is not equivalent to 1 on X, there exists a compact subset Y of X on which f is irreducibly not equivalent to 1. Further, any such set Y is a continuum not separated by any proper subcontinuum. Hence the property of having $f\sim 1$ is E_0-extensible for compact sets X; that is, if $f\sim 1$ on each E_0-set of every component of X, then $f\sim 1$ on X. In particular, property (b): to have every continuous mapping into the circle equivalent to 1, is E_0-extensible and reducible. Using this result it is shown directly that the number $b(Y)$ is cyclicly additive for compact sets X, that is, $b(X)=\sum b(E_\alpha)$ where the E_α are the simple links of the components of X and where, in general, $b(Y)$ denotes the rank of the factor group $S^Y/P(Y)$ where S^Y is the group of all mappings into S and $P(Y)$ is the subgroup of all elements of S^Y which are equivalent to 1 on Y. (Received January 21, 1941.)

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